Worksheet 21 (March 29)

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$A_{m\times n}$.

then $\mathrm{Row}(A) \subset \mathbb{R}^n$

$\mathrm{Nul}(A) \subset \mathbb{R}^n$

$\boxed{\mathrm{Nul}(A) = \mathrm{Row}(A)^\perp}$

$W = \{x+y+z=0\}$
$\quad \|$
$\mathrm{Nul}\,A$ for $A = (1\ 1\ 1)$

$W^\perp = \mathrm{Span}\left\{\begin{pmatrix}1\\1\\1\end{pmatrix}\right\}$.
$\quad \|$
$\mathrm{Row}\,A$.

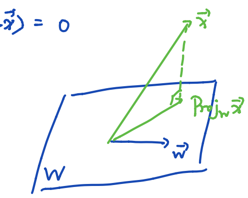
1 Problems

WTS $\vec{x}\cdot\vec{w} = 0$, $\forall \vec{w} \in W$.

In fact, $\vec{w} = c_1\vec{a_1} + c_2\vec{a_2} + \cdots + c_k\vec{a_k}$.

$\vec{x}\cdot\vec{w} = c_1(\vec{a_1}\cdot\vec{x}) + c_2(\vec{a_2}\cdot\vec{x}) + \cdots + c_k(\vec{a_k}\cdot\vec{x}) = 0$

Example 1. True or false.



(T) Let $W \subset \mathbb{R}^n$ be a subspace and $\mathbf{a}_1, \ldots, \mathbf{a}_k$ spans W. If $\mathbf{x}$ is orthogonal to each $\mathbf{a}_i (i = 1, 2, \ldots, k)$, then $\mathbf{x} \in W^\perp$.

(T) Let $W \subset \mathbb{R}^n$ be a subspace, $\mathbf{w} \in W$ and $\mathbf{x} \in \mathbb{R}^n$, then $(\mathrm{Proj}_W \mathbf{x}) \cdot \mathbf{w} = \mathbf{x} \cdot \mathbf{w}$.

(F) The orthogonal complement of $\underset{}{\overset{\subset \mathbb{R}^m}{\mathrm{Col}(A)}}$ is the $\overset{\subset \mathbb{R}^n}{\text{solution set}}$ of $\underset{m\times n}{A}\mathbf{x} = \mathbf{b}$.

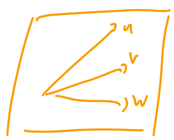
$\vec{x} = \mathrm{Proj}_W \vec{x} + \underset{\in W^\perp}{\vec{z}}$

$\vec{x}\cdot\vec{w} = (\mathrm{Proj}_W\vec{x})\cdot\vec{w} + \cancel{\vec{z}\cdot\vec{w}}$

Example 2. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be three unit vectors in $\mathbb{R}^n$ satisfying $\mathbf{u} + \mathbf{v} + \mathbf{w} = \mathbf{0}$. Prove that $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{u} = -1/2$.

$\vec{u} + \vec{v} = -\vec{w}$

unit vectors



WTS $\vec{u}\cdot\vec{v} = -1/2$

Idea Produce $\vec{u}\cdot\vec{v}$ out of $\vec{u}+\vec{v}$.

Details $(\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) = 1$

$\vec{u}\cdot\vec{u} + \vec{v}\cdot\vec{v} + 2\vec{u}\cdot\vec{v} = 2 + 2\vec{u}\cdot\vec{v} \rightsquigarrow \vec{u}\cdot\vec{v} = -\frac{1}{2}$.

Example 3. Find the orthogonal complement of the subspace $W \subset \mathbb{R}^3$ spanned by

$$\mathbf{w}_1 = \begin{pmatrix}2\\3\\4\end{pmatrix}, \mathbf{w}_2 = \begin{pmatrix}-1\\-1\\-2\end{pmatrix}.$$

Example 4. Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors in $\mathbb{R}^3$ whose orthongonal projection to the subspace W are $(1, 1, -1)^T$ and $(2, -4, 1)^T$ respectively.
(a) What is the orthogonal projection of $\mathbf{u} + 2\mathbf{v}$ to W?
(b) What is the smallest possible value of $\|\mathbf{u} - \mathbf{v}\|$?
(c) What is W?