Example e-values of A: 5 (alg. mult. $=2$), 3, 4.

then $\det A = 5^2 \cdot 3 \cdot 4$.

Worksheet 18 (March 15)

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$\det(A) = \cancel{\det(P^{-1})} \det(B) \cancel{\det(P)}$

Rmk. Corollary:

know $A = P^{-1} \cdot B \cdot P$ for P invertible.

1 Problems

→ $\det A = \det B$. if A & are similar.

$\chi_A(\lambda) = \det(A - \lambda I)$

$\chi_B(\lambda) = \det(B - \lambda I)$.

Example 1. True or false.

→ $\det A$ = product of all e-values of A with alg. mult.

$A - \lambda I = P^{-1} B P - P^{-1} \cdot \lambda I \cdot P = P^{-1}(B - \lambda I) \cdot P$

- (T) If A and B are similar, $\chi_A(\lambda) = \chi_B(\lambda)$.
- (F) If A and B are similar, then an eigenvector of A also an eigenvector of B.

$\det(A - \lambda I) = \cancel{\det(P^{-1})} \det(B - \lambda I) \cdot \cancel{\det(P)}$ ✓

- (T) If A is a 6×6 matrix which has 3 distinct eigenvalues with geometric multiplicities 3, 2 and 1, then A is diagonalizable. $\Leftrightarrow \sum$ geo. mult. $= n = 6$
- ( ) If square matrices A, P and D satisfies $AP = PD$ and D is diagonal, then nonzero columns of P are eigenvectors of A.

Example 2. Compute

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n.$$

The result may be used to compute the general formula of the Fibonacci sequence F_n, defined by $F_n = F_{n-1} + F_{n-2}$ and $F_1 = F_2 = 1$, but we will not discuss this application here.

Example 3. Let A and B be two $n \times n$ matrices such that $AB = BA$ and A has n distinct eigenvalues. Let $\mathbf{v} \in \mathbb{R}^n$ be an eigenvector of A. Prove that $\mathbf{v}$ is also an eigenvector of B.