Week 1 (1/19-1/22) Worksheet 1

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1 Review

DEFINITIONS

- linear equation, linear system;
- coefficient, $\mathbb{R}$ and $\mathbb{C}$, variable;
- solution, solution set, descriptive form of solution sets;
- consistent/inconsistent linear systems, equivalent linear systems.

2 Problems

Example 1. Solve the following linear systems

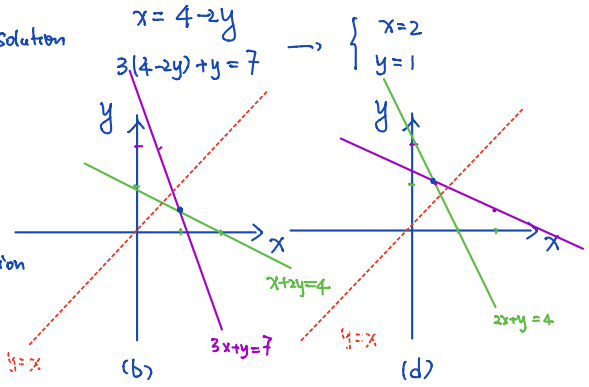
(a) $x = 3.$

(b) $\begin{cases} x + 2y = 4 \\ 3x + y = 7 \end{cases}.$

(c) $\begin{cases} x + 2y = 3 \\ 3x + 6y = 4 \end{cases}.$

(d) $\begin{cases} 2x + y = 4 \\ x + 3y = 7 \end{cases}.$

(e) $\begin{cases} x_1 + x_2 + x_3 = 3 \\ x_1 + 2x_2 + 3x_3 = -1 \\ x_1 + 3x_2 + 5x_3 = -5 \end{cases}.$



Example 2. Think about the following questions

- Among the linear systems in 1, which are consistent? Inconsistent? Are there equivalent linear systems?
- Compare the solution sets in (b) and (d), what do you see, and why?

Example 3. Find the value of c such that the following system is inconsistent

$$\begin{cases} x_1 + cx_2 = -1 & ① \\ 2x_1 - 2x_2 = 0 & ② \end{cases}.$$

Example 4. Find the value of the coefficient c such that the following two systems are equivalent

$$\begin{cases} x_1 - cx_2 = 0 \\ x_1 + x_3 = 0 \end{cases}, \qquad \begin{cases} 2x_1 - x_2 + x_3 = 0 \\ x_2 + x_3 = 0 \end{cases}.$$

A B

Example 3 Method ① inconsistent $\Leftrightarrow$ "Contradictions" among linear equations

e.g. $\begin{cases} x+y+z = 1 \\ x+y = 2 \\ 2x+2y+z = 4 \end{cases}$

$x_1 - x_2 = 0$

take $c=-1$ ① $\rightsquigarrow$ $x_1 - x_2 = -1$

Method ② $\begin{bmatrix} 1 & c & -1 \\ 2 & -2 & 0 \end{bmatrix}$ reduce to RREF

last column is pivot. solve c.

Example 4 ① Solution set of B is $\{(-a, -a, a) \mid a \in \mathbb{R}\}$

of A is $\{(-a, \frac{1}{c}a, a) \mid a \in \mathbb{R}\}$ $c = +1$

② RREF equivalent systems have the same RREF.