

Week 1 (1/19-1/22) Worksheet 1

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1 Review

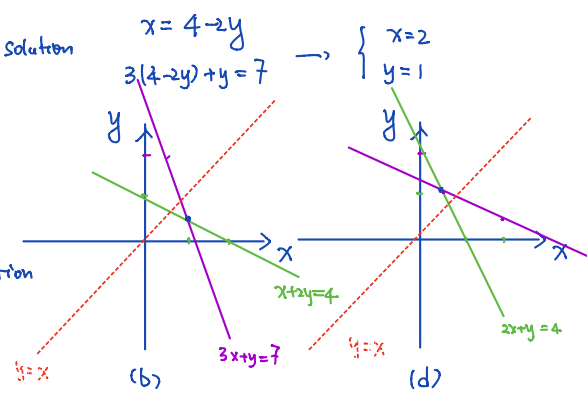
DEFINITIONS

- linear equation, linear system;
 - coefficient, \mathbb{R} and \mathbb{C} variable;
 - solution, solution set, descriptive form of solution sets;
 - consistent/inconsistent linear systems, equivalent linear systems.
- \mathbb{Q} rational numbers \mathbb{R} real numbers \mathbb{C} complex numbers $\sqrt{-1}=i$ unknowns
 all solutions values of variables solutions exist (≥ 1) (0 solution)
- $(a_1)x_1 + (a_2)x_2 + \dots + (a_n)x_n = b$
 $\mathbb{C} = \{a+bi \mid a, b \in \mathbb{R}\} \supset \mathbb{R}$
 $r \cdot e^{i\theta}$ $r = \sqrt{a^2+b^2}$ $\theta = \arctan \frac{b}{a}$ $\tan \theta = \frac{b}{a}$
- e.g. $3x_1 + 4x_2 + 5x_3 = 20$
 $x_1 = 4 - x_2 - x_3$
 $x_1 \cdot x_2$
- Solution set of e.g. $\{(20 - 4a - 5b, a, b) \mid a, b \in \mathbb{R}\}$
 have the same solution set. e.g. $6x_1 + 8x_2 + 10x_3 = 40$
 $\Leftrightarrow 3x_1 + 4x_2 + 5x_3 = 20$

2 Problems

Example 1. Solve the following linear systems

- (a) $x = 3$. $\{x=3\}$, $\{3\}$ unique solution
- (b) $\begin{cases} x + 2y = 4 \\ 3x + y = 7 \end{cases}$. $\{(x,y) = (2,1)\}$. $\{(2,1)\}$ unique solution
- (c) $\begin{cases} x + 2y = 3 \\ 3x + 6y = 4 \end{cases}$. \emptyset empty set no solution
- (d) $\begin{cases} 2x + y = 4 \\ x + 3y = 7 \end{cases}$. $\{(x,y) = (1,2)\}$ $\{(1,2)\}$ unique solution
- (e) $\begin{cases} x_1 + x_2 + x_3 = 3 \\ x_1 + 2x_2 + 3x_3 = -1 \\ x_1 + 3x_2 + 5x_3 = -5 \end{cases}$



Example 2. Think about the following questions

- Among the linear systems in 1, which are consistent? Inconsistent? Are there equivalent linear systems?
- Compare the solution sets in (b) and (d), what do you see, and why?

Example 3. Find the value of c such that the following system is inconsistent

$$\begin{cases} x_1 + cx_2 = -1 & \textcircled{1} \\ 2x_1 - 2x_2 = 0 & \textcircled{2} \end{cases}$$

Example 4. Find the value of the coefficient c such that the following two systems are equivalent

$$\begin{cases} x_1 - cx_2 = 0 \\ x_1 + x_3 = 0 \end{cases} \quad \text{A} \qquad \begin{cases} 2x_1 - x_2 + x_3 = 0 \\ x_2 + x_3 = 0 \end{cases} \quad \text{B}$$

Example 3 Method ① inconsistent \Leftrightarrow "Contradictions" among linear equations

e.g. $\begin{cases} x+y+z=1 \\ x+y=2 \\ 2x+2y+z=4 \end{cases}$

$x_1 - x_2 = 0$

take $c = -1$. ① $\leadsto x_1 - x_2 = -1$

Method ② $\begin{bmatrix} 1 & c & -1 \\ 2 & -2 & 0 \end{bmatrix}$ reduce to RREF
last column is pivot. solve c .

Example 4 ① Solution set of B is $\{(-a, -a, a) \mid a \in \mathbb{R}\}$
of A is $\{(-a, \frac{1}{c}a, a) \mid a \in \mathbb{R}\}$ $(c = +1)$

② RREF equivalent systems have the same RREF.