How mathematicians can contribute to K–12 mathematics education

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"To overcome the isolation of education research, more effective links must be created between educational faculties and the faculties of universities. This could allow scholars of education better acquaintance with new developments in and across the disciplines and other professional fields of the university, while also encouraging discipline-based scholars with interests in education to collaborate in the study of education."
Lagemann, 2000, p. 241.

I would like to make a general disclaimer at the outset. I think I should only talk about things I know firsthand, so I will limit my comments to the K–12 mathematics education in the U.S. rather than take a more global view. Such a restriction is not necessarily fatal since a friend of mine observed that what takes place in the U.S. tends also to take place elsewhere a few years later. For example, in France there is now a Math War that resembles the American Math Wars of the nineties (Education Week, 2005). We live in a global village after all.

Let me begin with a fairy tale. Two villages are separated by a hill, and it was decided that for ease of contact, they would drill a tunnel. Each village was entrusted with the drilling of its own half of the tunnel, but after both had done their work, it was discovered that the two halves didn't meet in the middle of the hill. Even though a connecting tunnel between the two lengths already built could be done at relatively small expense, the two villages, each in defense of its honor, prefer to continue the quarrel to this day.

This fairy tale is too close to reality for comfort when the two villages are replaced by the education and mathematics communities, with the former emphasizing the overriding importance of pedagogy and the latter, mathematical content.[1] Mathematics education rests on the twin pillars of mathematics and pedagogy, but the ongoing saga in mathematics education is mostly a series of episodes pitting one against the other. There is probably no better proof of the disunity between these communities than the very title of this article. Indeed, if someone were to write about "How chemists can contribute to chemical engineering", that person would be considered a crank for wasting ink on a non-issue. Chemical engineering is a well-defined discipline, and chemical engineers are perfectly capable of doing what they are entrusted to do. They know the chemistry they need for their work, and if there is any doubt, they would freely consult with their colleagues in chemistry in the spirit of cooperation and collegiality. Therefore, the fact that we are going to discuss "How mathematicians can contribute to K–12 mathematics education" in the setting of the International Congress speaks volumes about both mathematics education and mathematicians.

The title of this article implicitly gives away the power structure of mathematics education in the academic world: Educators hold the rein. Since education research is thriving and research funding is ample, it is not surprising that educators want to protect their intellectual independence in the university environment. Rumblings about how mathematically unqualified teachers or deficient curricula are undercutting mathematics learning do surface from time to time, but we have not witnessed the expected aggressive action agitating for collaboration with mathematicians. Other troubling issues related to mathematics content, such as the presence of incorrect assessment items in standardized tests, likewise fail to arouse genuine concern in the mathematics education community. To an outsider, the protection of the "education"

[1] In writing about sociological phenomena, especially education, it is understood that all statements are statistical in nature unless stated to the contrary, and that exceptions are part and parcel to each statement. In fact, there are striking (though isolated) exceptions in the present context. The reader is asked to be aware of this *caveat* for the rest of this article.

enclave seems to matter more to university educators than collaboration with the research mathematics community that could strengthen K–12 mathematics education. By contrast, if the department of chemical engineering consistently produces engineers with a defective knowledge of chemistry, or if accidents occur in its laboratories with regular frequency, would the chemical engineering faculty not immediately spring to action? This question prompts the thought that maybe we no longer know what mathematics education is about, and it is time for us to take a second look.

One meaning of the word "engineering" is the art or science of customizing scientific theory to meet human needs. Thus chemical engineering is the science of customizing chemistry to solve human problems, or electrical engineering is the science of customizing electromagnetic theory to design all the nice gadgets that we have come to consider indispensable. I will put forth the contention that mathematics education is **mathematical engineering**, in the sense that it is the customization of basic mathematical principles to meet the needs of teachers and students.[2] I will try to convince you that this is a good model for the understanding of mathematics education before proceeding to a discussion of how mathematicians can contribute to K–12 mathematics education. The far-from-surprising conclusion is that, unless mathematicians and educators can work as equal partners, K–12 mathematics education cannot improve.

Regarding the nature of mathematics education, Bass (2005) made a similar suggestion that it should be considered a branch of applied mathematics.[3] What I would

[2]After the completion of this article, Skip Fennell brought to my attention the article "Access and Opportunities to Learn Are Not Accidents: Engineering Mathematical Progress in Your School" by William F. Tate, which is available at:
http://www.serve.org/_downloads/publications/AccessAndOpportunities.pdf
Tate is concerned with equity and uses "engineering" as a metaphor to emphasize the potential for designing different educational policies and pedagogical activities to promote learning, but without addressing the mathematics. On the other hand, the present article explains why mathematics education *is* the engineering of mathematics.

[3]Hy Bass lectured on this idea in December of 1996 at MSRI, but [5] seems to be a convenient reference. After the completion of this article, Zalman Usiskin informed me that in the Proceedings of the U.S.-Japan workshop on the mathematics education of teachers in 2000 that followed ICME-9 in Japan, he had written that "'Teachers' mathematics' is a field of applied mathematics that deserves its own place in the curriculum." Along this line, let it be mentioned that the paper of Ferrini-Mundy and Findell [8] made the same assertion and, like Bass, it does not touch on the engineering aspect of mathematics education. The need for mathematicians and educators to work

like to emphasize is the aspect of engineering that *customizes* scientific principles to the needs of humanity in contrast with the scientific-application aspect of applied mathematics. Thus, when H. Hertz demonstrated the possibility of broadcasting and receiving electromagnetic waves, he made a breakthrough in science by making a scientific application of Maxwell's theory. But when G. Marconi makes use of Hertz's discovery to create a radio, Marconi was making a fundamental contribution in electrical *engineering*, because he had taken the extra step of harnessing an abstract phenomenon to fill a human need.[4] In this sense what separates mathematics education as mathematical engineering from mathematics education as applied mathematics is the crucial step of *customizing* the mathematics, rather than simply applying it in a straightforward manner to the specific needs of the classroom. There is no better illustration of this idea of customization than the teaching of fractions in upper elementary and middle schools, as I now explain.

Students' failure to learn fractions is well-known. School texts usually present a fraction as parts of a whole, i.e., pieces of a pizza, and this is the most basic conception of a fraction for most elementary students. However, when fractions are applied to everyday situations, then it is clear that there is more to fractions than parts-of-a-whole, e.g., if there are 15 boys and 18 girls in a classroom, then the ratio of boys to girls is the fraction $\frac{15}{18}$, which has nothing to do with cutting up a pizza into 18 equal parts and taking 15. In the primary grades, it is not a serious problem if students' knowledge of fractions is imprecise and informal, so that a fraction can be simultaneously parts-of-a-whole, a ratio, a division, and an operator[5], and a number. Children at that age are probably not given to doubts about the improbability of an object having so many wondrous attributes. At some stage of their mathematical development, however, they will have to make sense of these different "personalities" of a fraction. It is this transition from intuitive knowledge to a more formal and abstract kind of mathematical knowledge that causes the most learning problems. This transition usually takes place in grades 5–7.

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on equal footing in mathematics education is likewise not mentioned by these educators.

[4]The invention was actually due to N. Tesla, but like many things in life, popular preception displaces the truth. I am indebted to S. Simic for pointing this out to me.

[5]For example, the fraction $\frac{3}{4}$ can be regarded as a function (operator) which associates to each quantity three-quarters of the same quantity.

There is by now copious mathematics education research[6] on how to facilitate children's learning of the fraction concept at this critical juncture in order to optimize their ability to use fractions efficiently. At present, what most children get from their classroom instruction on fractions is a fragmented picture of a fraction with all these different "personalities" lurking around and coming forward seemingly randomly. What a large part of this research does is to address this fragmentation by emphasizing the *cognitive* connections between these "personalities". It does so by helping children construct their intuitive knowledge of the different "personalities" of a fraction through the use of problems, hands-on activities, and contextual presentations.

This is a good first step, and yet, if we think through students' mathematical needs beyond grade 7, then we may come to the conclusion that establishing cognitive connections does not go far enough. What students need is an unambiguous *definition* of a fraction which tells them what a fraction really *is*. They also need to be exposed to *direct, mathematical,* connections between this definition and the other "personalities" of a fraction. They have to learn that mathematics is *simple and understandable*, in the sense that if they can hold onto one clear meaning of a fraction and can reason for themselves, then they can learn all about fractions without ever being surprised by any of these other "personalities".

From a mathematician's perspective, this scenario of having to develop a concept with multiple interpretations is all too familiar. In college courses, one approaches rational numbers (both positive and negative fractions) either abstractly as the prime field of characteristic zero, or as the field of quotients of the integers. The problem is that *neither is suitable for use with fifth graders.* This fact is recognized by mathematics education researchers, as is the fact that from such a precise and abstract definition of rational numbers, one can *prove* all the assorted "personalities" of rational numbers. If I have read the research literature correctly, these researchers despair of ever being able to offer proofs once they are forced to operate without an abstract definition, and that is why they opt for establishing cognitive, rather than mathematical connections among the "personalities" of rational numbers. The needs

[6] Here as elsewhere, I will not supply explicit references because I do not wish to appear to be targeting specific persons or works in my criticism. I will be making *generic* comments about several general areas.

of the classroom would seem to be in conflict with the mathematics. At this point, engineering enters.

It turns out that, by changing the mathematical landscape entirely and leaving quotient fields and ordered pairs behind, it is possible to teach fractions *as mathematics* in elementary school, by finding *an alternate mathematical route around these abstractions that would be suitable for consumption by children in grades 5–7.* Without going into details, suffice it to say that at least the *mathematical* difficulties can be overcome, for example, by identifying fractions with certain points on the number line (for this systematic development, see, e.g., Jensen 2003, or Wu, 2001c). What is of interest in this context is that this approach to fractions is specific to the needs of elementary school and is not likely to be taught, ever, in any other situation. In addition, the working out of the basic properties of fractions from this viewpoint is not quite straightforward, and it definitely requires the expertise of a research mathematician. As to the further pedagogical implementation to render such an approach usable in grades 5–7, the input of teachers and educators would be absolutely indispensable.[7] We therefore get to witness how mathematicians and educators are both needed to turn a piece of abstract mathematics into usable lessons in the school classroom. This is customization of abstract theory for a specific human need, and this is engineering at work.

Through this one example of fractions, we get a glimpse of how the principles of mathematical engineering govern the design of a curriculum. Less obvious but of equal importance is the fact that even mathematics education research cannot be disconnected from the same principles. If, for example, a strong mathematical presence had been integral to the research on fractions and rational numbers, it would be very surprising that the research direction would have developed in the direction it did. Compare the quote by Lagemann at the beginning of this article as well as Lagemann, 2000.

An entirely analogous discussion of customization can be given to any aspect of mathematics education, but we single out the following for further illustrations:

(a) The design of an "Intervention Program" for at-risk students. Up to this

[7]Some teachers who have worked with me are trying out this approach with their students in San Francisco.

point, the methods devised to help these students are largely a matter of teaching a watered-down version of each topic at reduced pace; this is poor engineering from both the theoretical and the practical point of view. In Milgram-Wu, 2005, a radically different mathematical engineering design is proposed to deal with this problem.

(b) The teaching of beginning algebra in middle school. The way symbols are usually handled in such courses, which necessitates prolix discussions in the research literature of the subtlety of the *equal sign*, and the way *variable* is introduced as the central concept in school algebra are clear indications that the algebra we teach students at present has not yet been properly customized for the needs of school students. See the Preface and Sections 1 and 2 of Wu, 2005d, and also Wu, 2006, for a more detailed account of both the problems and their proposed solutions.

(c) The writing of mathematics standards at the national or state level. This is an example of what might be called "practical optimization problems", which customize the mathematics to meet diverse, and at times conflicting, needs of different clientele. Cf. Klein et al., 2005.

The concept of mathematics education as mathematical engineering also sheds some light on Lee Shulman's (1986) concept of *pedagogical content knowledge*. There has been a good deal of interest in precisely describing the kind of knowledge a teacher should possess in order to be effective in teaching. In the field of mathematics, at least, this goal has proven to be elusive thus far (but cf. Hill-Rowan-Ball, 2004), but Shulman's intuitive and appealing formulation of this concept crystallizes the diverse ideas concerning an essential component of good teaching. From the point of view of mathematical engineering, one of the primary responsibilities of a teacher is to customize her mathematical knowledge in accordance with the needs of each situation for students' consumption. This particular engineering knowledge is the essence of pedagogical content knowledge. Although this approach to pedagogical content knowledge does not add anything new to its conception, it does provide a framework to understand this knowledge within mathematics, one that is different from what one normally encounters in educational discussions. It makes explicit at least three components to effective teaching: a solid mathematical knowledge, a clear perception of the setting defined by the students' knowledge, and the flexibility of mind to customize this mathematical knowledge for use in this particular setting without sacrificing mathematical integrity.

The idea of customizing mathematics "without sacrificing mathematical integrity" is central to mathematical engineering. In engineering, it is obvious that, in trying to customize scientific principles to meet the needs of humanity, we cannot contradict nature regardless of how great the human needs may be. In other words, one respects the integrity of science and does not attempt anything so foolish as the construction of anti-gravity or perpetual-motion machines. Likewise, as mathematical engineering, mathematics education accepts the centrality of mathematics as a given. Again using the example of teaching fractions, a mathematics educator would know that no matter how one tries to teach fractions, it must be done in a way that respects the abstract meaning of a fraction even if the latter is never used explicitly. If, for instance, an educator catches himself saying that children must adopt new rules for fractions that often conflict with well-established ideas about whole numbers, then he knows he is teaching fractions the wrong way because, *no matter what efforts one puts into making fractions intuitive to children, one cannot do violence to the immutable fact that the rational numbers contain the integers as a sub-ring.* The need to teach the arithmetic of fractions as a natural extension of the arithmetic of whole numbers has gone unnoticed for far too long, with the result that too many of our students begin to harbor the notion that, after the whole numbers, the arithmetic of fractions is a *new beginning.* Such bad mathematical engineering in curricular designs is unfortunately a common occurrence.

The only way to minimize such engineering errors is to have both mathematicians and educators closely oversee each curricular design. In fact, if we believe in the concept of mathematics education as mathematical engineering, then the two communities *must* work together in all phases of mathematics education: Any education project in mathematics must begin with a sound conception of the *mathematics* involved, and there has to be a clear understanding of what the *educational* goal is before one can talk about customization. In this process, there is little that is purely mathematical or purely educational; almost every step is a mixture of both. Mathematics and education are completely intertwined in mathematical engineering.

Mathematicians cannot contribute to K–12 mathematics education if they are treated as outsiders.[8] *They have to work alongside the educators on equal footing* in the planning, implementation, and evaluation of each project. But this is far from the reality at present.

For at least three decades now, the mathematics and K–12 education communities in the U.S. have not been on speaking terms in the figurative sense. (Cf. Washington Post, 1999.) The harm this communication gap has brought to K–12 mathematics education can be partially itemized, but before doing that, let me point out three general consequences of a philosophical nature. The first one is that the isolation of the education community from mathematicians causes educational discussions to over-focus on the purely education aspect of mathematics education while seemingly always leaving the mathematics untouched. The result is the emergence of a subtle *mathematics avoidance syndrome* in the education community, and this syndrome will be seen to weave in and out of the following discussion of the specific harmful effects of this communication gap. Given the central position of mathematics in mathematical engineering, it would be noncontroversial to say that this syndrome should vanish from all discussions in mathematics education as soon as possible.

The fact that many mathematicians *teach* mathematics and *design* mathematics courses throughout their careers seems to escape the attention of many educators. Here is a huge reservoir of knowledge and experience in mathematical engineering on tap. The chasm between the two communities in effect denies educators access to this human resource at a time when educators need all the engineering help they can get.

The final consequence can best be understood in terms of the Darwinian dictum that when a system is isolated and allowed to evolve of its own accord, it will inevitably mutate and deviate from the norm. Thus when school mathematics education is isolated from mathematicians, so is school mathematics itself, and, sure enough, the latter evolves into something that in large part no longer bears any resemblance to mathematics. Correct definitions are not given, or if given, they are not put to use (Milgram-Wu, 2005, Wu, 2001a, 2005a and 2005c). The organic co-

[8]This only tells half the story about mathematicians. See the comments near the end of this article.

herence of mathematics is no longer to be found (Wu, 2002), or when "mathematical connections" are intentionally emphasized, such "connections" tend to be the trivial and obvious kind. Logical deduction becomes an afterthought; proofs, once relegated to the secondary school geometry course, were increasingly diluted until by now almost no proofs at all are found there, or anywhere else in the schools (Wu, 2004). And so on. This development naturally brings down the quality of many aspects of mathematics education.

The absence of dialog between the two communities has led to many engineering errors in mathematics education, one of them being the unwelcome presence of mathematically incorrect test items in state and other standardized tests (Milgram 2002, 2003). The same kind of defective items also mar many teachers' credentialing tests (Askey 2006a, 2006b). A more subtle effect of the absence of mathematical input on assessment is the way test scores are routinely misinterpreted. The low test scores have been used to highlight students' dismal mathematical performance, but little or no thought is given to the possibility that they highlight not necessarily students' achievement (or lack thereof) but the pervasive damage done by defective curricular materials, or even the chronic lack of effective teaching. Such a possibility may not be obvious to anyone outside of mathematics, but to a mathematician, it does not take any research to confirm the fact that when students are taught incorrect mathematics, they learn incorrect mathematics. *Garbage in, garbage out.* If the incorrect mathematics subsequently shows up in students' test scores, how can we separate the errors due to the incorrect information students were given, from the errors due to students' own misconceptions? A more detailed examination of this idea in the narrow area of school algebra is given in Wu, 2006. The need for mathematicians' participation in all phases of assessment is all too apparent.

The lack of collaboration between mathematicians and mathematics educators affects professional development as well. The issue of teacher quality is now openly acknowledged and serious discussions of the problem are beginning to be accepted in mathematics education (cf. Ma, 1999, and Conference Board of the Mathematical Sciences, 2001[9]). As a result of the inadequate mathematics instruction teachers receive in K–12, their knowledge of mathematics is, by and large, the product of the

[9]Whatever reservations one may have concerning the details of its content, it is the fact that such a volume could be published under the auspices of a major scientific organization that is important.

mathematics courses they take in college.[10] In very crude terms, the number of such required mathematics courses is too low, and in addition, these courses are taught either by mathematicians who are not in close consultation with teachers, and are unaware as to what is needed in the school classroom, or by mathematics educators who are not professional mathematicians. The former kind of course tends to be irrelevant to the classroom, and the latter kind tends to be mathematically shallow or incorrect. It is only natural that teachers coming out of such an environment turn out to be mathematically ill-prepared.

Similar woes persist in in-service professional development, thereby ensuring that teachers have little access to the mathematical knowledge they need for their profession. For example, the last decade has witnessed the appearance of *case books* consisting of actual records of lessons given by teachers.[11] The idea is to invite teachers to analyze these lessons, thereby sharpening their pedagogical sensibilities. In too many instances, however, blatant mathematical flaws in the cited cases are overlooked in the editors' commentaries. This raises the specter of bringing up a generation of teachers who are proficient in teaching school students *incorrect* mathematics. In this instance, it would appear that the need to respect mathematical integrity in mathematical engineering has been all but forgotten.

The most divisive outcome of the noncommunication between the two communities in the U.S. is undoubtedly the conflict engendered by the new (reform) curricula written in the past fifteen years. I take up this discussion last, because it brings us face to face with some subtle issues about mathematicians' participation in K–12 mathematics education. The prelude to the writing of these curricula is the unchecked degeneration in the mathematical integrity of the existing textbooks from major publishers over the period 1970-1990, a fact already alluded to above. This degeneration triggered the reform spearheaded by NCTM (National Council of Teachers of Mathematics, 1989). Rightly or wrongly, the new curricula were written under the banner of the NCTM reform, and the manner in which some of the reform texts were imposed

[10] It may be useful to also take note of what may be called "the second order effect" of university instruction: teachers' knowledge of mathematics is also conditioned by their own K–12 experiences, but these teachers' teachers were themselves products of the mathematics courses they took in the university.

[11] Let it be noted explcitly that I am discussing the case books in K–12 mathematics education only.

on public schools led eventually to the well-known Math Wars (Jackson, 1997). The root of the discontent over these texts is the abundance of outright mathematical errors,[12] as well as what research mathematicians perceived to be evidence of a lack of understanding of the mathematics. An example of the latter was the promotion of children's *invented algorithms* at the expense of the standard computation algorithms in the elementary mathematics curriculum. Although the promotion was partly an overreaction to the way the standard algorithms were often inflicted on school children with nary a word of explanation, it also reflected a lack of awareness of the central importance of the mathematical lessons conveyed by the reasoned teaching of these algorithms.

The "subtle issues" mentioned above stem from the fact that the writing of some of the new reform curricula actually had the participation of a few mathematicians. The first thing to note is that the latter *are* the rare exceptions to the general noncommunication between the mathematics and education communities. The noncommunication is real. At the same time, these exceptions seem to point to an apparent contradiction: How would I reconcile my critical stance toward these reform curricula with the principal recommendation of this article, namely, that mathematicians be equal partners with educators in the mathematics education enterprise? The answer is that there is no contradiction at all. The participation by mathematicians is, in general terms, a *prerequisite* to any hope of success in K–12 mathematics education, but in no way does it guarantee success. It is helpful in this context to recall similar discussions that routinely took place some eight years ago when some mathematicians first went public with the idea that mathematics teachers must have a solid content knowledge. The usual rejoinder at the time was that "knowing mathematics is not enough (to be a good teacher)". This is a common confusion that mistakes a necessary condition for a sufficient condition.[13] There is no quick fix for something as complex as mathematics education. Getting mathematicians to fully participate is only the beginning; the choice of the mathematicians and the hard work to follow will have a lot to say about the subsequent success or failure.

It is appropriate at this point to recall what was said at the beginning of the

[12]These errors tend to be different from the earlier ones to be sure, but errors they are.

[13]And need I point out, there are some who intentionally use this confusion to reject that mathematical content knowledge is important for teachers, or that getting mathematicians to participate in mathematics education is critical for its success.

article about the power structure of mathematics education: thus far, educators get to make the decisions. Granting this fact, I should amplify a bit on the difficulties of choosing the right mathematicians for education work. Mathematicians have a range of background and experiences and, consequently, often have a range of opinions on matters of education as well. It is important that *the range of these opinions be considered* in all aspects of education. Many of the less happy incidents of the recent past in K–12 mathematics education were the result of choosing mathematicians of a particular persuasion. In addition, educators must make their own judgement on which among the mathematicians interested in K–12 are knowledgeable about K–12. Among the latter, some possess good judgment and leadership qualities while others don't. Educators must choose at each step. If there are algorithms for making the right choices, I don't happen to know them.

Every mathematician potentially has something to offer in K–12 mathematics education: even an occasional glance at textbooks to check for mathematical correctness can be very valuable. However, if mathematicians want to participate in serious educational work in K–12, what must they bring to the table? I believe the most important thing is the awareness that K–12 mathematics education is *not* a subset of mathematics, and that there is quite a bit to learn about the process of *customization* that distinguishes K-12 mathematics education from mathematics. In particular, much (if not most) of the mathematics they teach in the university cannot be brought straight to the school classroom (Wu, 1997; Kilpatrick et al., 2001, Chapter 10 and especially pp. 375-6), but that it must first go through the engineering process to make it suitable for use in schools. If I may use the example of fractions once again, mathematicians interested in making a contribution to K–12 may find it instructive to get to know the reason that something like "equivalence classes of ordered pairs of integers" is totally opaque to students around the age of twelve. They would also want to know the reason that students of that age nonetheless need a definition of a fraction which is as close to parts-of-a-whole as possible. They should also get to know the appropriate kind of mathematical reasoning for students in this age group, because they will ultimately be called upon to safeguard such reasoning in the curriculum and assessment for these students.

Mathematicians may regard school mathematics as technically primitive (in the sense of skills), but they must take note of its conceptual sophistication (Jensen, 2003;

Wu, 2001b, 2001c, and 2005d; cf. also Aharoni, 2005). Above all, they must know that school mathematics is anything but *pedagogically* trivial: There is absolutely nothing trivial about putting any material, no matter how simple, into a correct mathematical framework so that it may be profitably consumed by school students. Mathematicians who want to contribute to K-12 mathematics education have to be constantly on the alert to ensure that the minimum requirements of their profession — the orderly and logical progression of ideas, the internal cohesion of the subject, and the clarity and precision in the presentation of concepts, — are still met in mathematics education writings. This is no easy task. If mathematicians want to enter K–12 mathematics education as equal partners with educators, then it is incumbent upon them to uphold their end of the bargain by acquiring this kind of knowledge about mathematical engineering.

The concept of mathematics-education-as-mathematical-engineering does not suggest the creation of any new tools for the solution of the ongoing educational problems. What it does is to provide a usable intellectual framework for mathematics education as a discipline, one that clarifies the relationship between the mathematics and the education components, as well as the role of mathematicians in mathematics education. For example, it would likely lead to a better understanding of why the New Math became the disaster that it did. Most importantly, this concept lays bare the urgent need of the mathematical presence in every aspect of K–12 mathematics education, thereby providing a strong argument against the self-destructive policy of keeping mathematicians as outsiders in mathematics education. The chasm between mathematicians and educators must be bridged if our children are to be better served. I am cautiously optimistic[14] that there are enough people who want to rebuild this bridge (cf. Ball et al., 2005), all the more so because the indications are that the NCTM leadership is also moving in the same direction. I look forward to a future where mathematics education is the joint effort of mathematicians and educators.

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[14]In January of 2006.

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