The Common Core Mathematics Standards: Implications for Administrators

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Many sets of state and national math standards have come and gone in the past twenty years.

By 2014, the Common Core Mathematics Standards (**CCMS**) will be phased in. Will it be just more of the same?

No.

At least among the better standards, change usually means reshuffling or wordsmithing the same collection of statements. If some standards are moved up to an earlier grade, then many would consider the new set of standards to be *more rigorous*.

In this metric, a set of *rigorous standards* is one in which each topic is taught as early as possible.

The underlying assumption is that **the mathematics of the school curriculum is set and done, and is beyond reproach,** so that all that remains for a set of standards to do is to package its many components judiciously.

The reality is different.

CCMS seems to be the first set of standards to be aware of this difference and address it head-on.

To some people, since the mathematics of the school curriculum is already in good shape, the main concerns of a new set of standards should be how to make it more *rigorous* and how to jazz up the mathematics so that students acquire “21st century skills”. CCMS concentrates instead on righting the wrongs in the mathematics of the existing curriculum.

There has not been any similar effort within memory.

Instead of engaging in the senseless game of acceleration—teaching each topic as early as possible—CCMS asks if we are properly preparing our students to learn the mathematics they need to learn.

It does not cram all of Algebra I into grade 8 in order to teach students the geometry they need for algebra. It mandates continuity in students' learning going from grade to grade.

Students can no longer forget what they learned the year before.

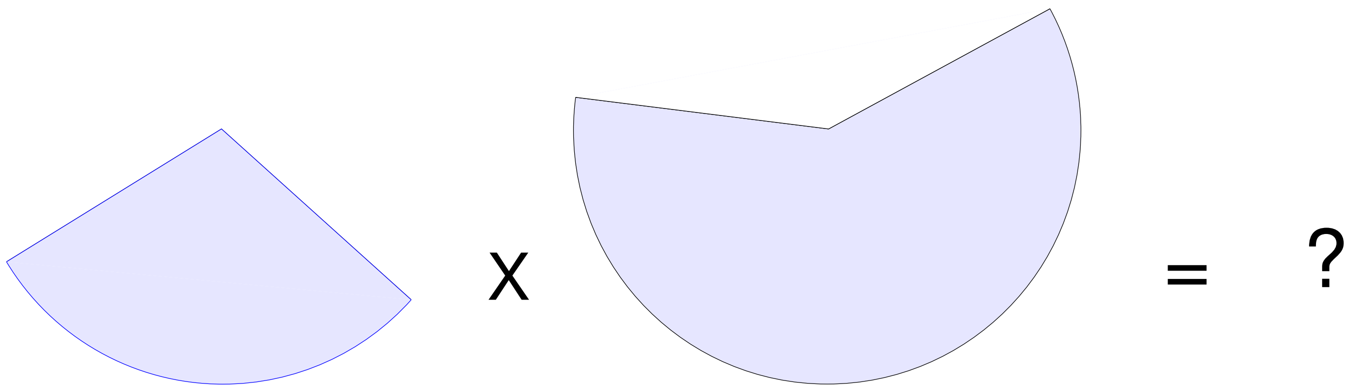
Getting the math right wins few stylistic points, but it is crucial for educational progress. If we don't get it right, our students cannot learn. **Garbage in, garbage out.**

You may have heard of the problem with *proofs in geometry*. You may have heard of *algebra being the unattained civil right.* You may have heard of *fraction-phobia*. All that because of **garbage in, garbage out.**

We as a nation have been suffering from this educational malaise for decades.

I want to give you some examples to illustrate the **reality** of what is going on in the school mathematics classroom.

(1) If a fraction is a piece of pie, how can we make students understand multiplying two pieces of pie?



$$\frac{2}{7} \times \frac{3}{5} = ?$$

(2) Solve: *Ann walks briskly and covers 3 miles the first hour. How many miles does she cover in 84 minutes?*

Set up proportion: Let Ann cover x miles in 84 minutes. Then 60 minutes is to 3 miles as 84 minutes is to x miles. So

$$\frac{60}{3} = \frac{84}{x}$$

Answer: $x = 4\frac{1}{5}$ miles.

Using the same reasoning, we do the following problem:

A stone is dropped from 144 ft. It drops 16 ft the first second. How much does it drop in 3 seconds?

If it drops x feet in 3 seconds, then 1 second is to 16 ft as 3 seconds is to x ft.

$$\frac{1}{16} = \frac{3}{x} \qquad \text{Ans: 48 feet.}$$

(Correct answer: 144 ft. It reaches the ground after 3 seconds.)

(3) Adding fractions.

To add $\frac{\mathbf{7}}{\mathbf{8}} + \frac{\mathbf{5}}{\mathbf{6}}$, take the **LCD** of 8 and 6, which is 24. Note that $24 = 3 \times 8$ and $24 = 4 \times 6$. Therefore

$$\frac{7}{8} + \frac{5}{6} = \frac{(3 \times 7) + (4 \times 5)}{24} = \frac{41}{24}$$

Adding is supposed to "combine things". The concept of "combining" is so basic that it is always taught at the beginning of arithmetic.

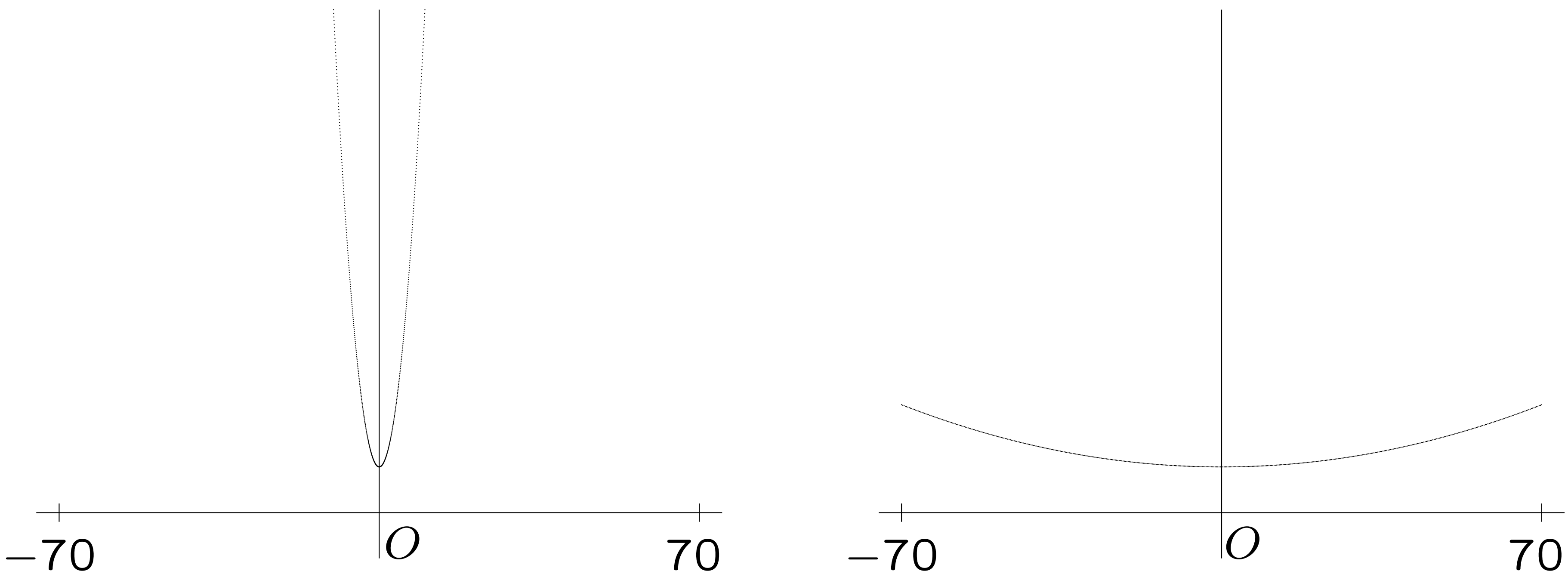
But did you see any "combining" in this addition?

(4) What is a parabola?

According to one algebra textbook book: *A* **parabola** *is the general shape of the graph of a quadratic function.*

According to another algebra textbook: *The graphs of quadratic functions all curve in a similar way. Such a graph is called a* **parabola.**

Now, do the following graphs “curve in a similar way”?



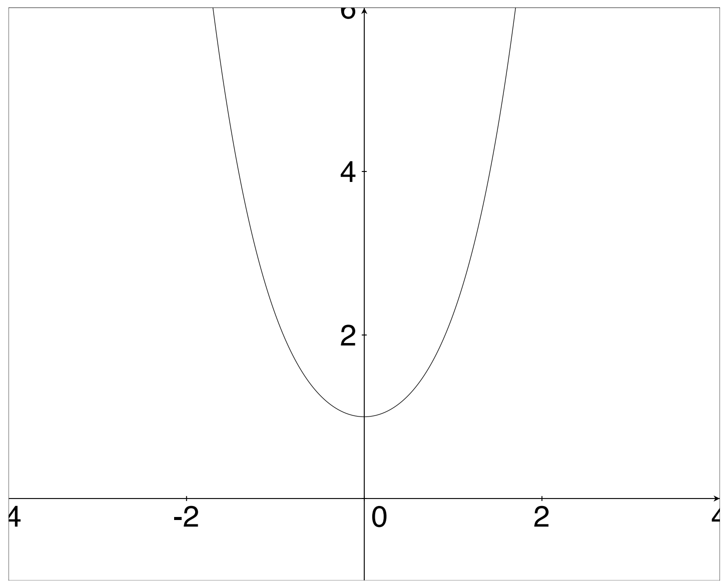
They may not look like it, but they are both graphs of quadratic functions. The left curve is the graph of

$$x^2 + 10$$

while the right curve is the graph of

$$\frac{1}{360}\,x^2 + 10$$

On the other hand, does the following curve "have the general shape of the graph of a quadratic function"?



You may think so, but this is not the graph of a quadratic function because it is the graph of $\frac{1}{4}x^4 + x^2 + 1$, which is definitely not quadratic.

So if you are trying to learn about parabolas from existing textbooks, what do *you* think is a "parabola"?

These examples serve to illustrate the quality of the mathematics that is encoded in our textbooks (there is not much difference between them). A perennial problem in school mathematics education has been this:

> The mathematics defined by school textbooks is too often inscrutable and beyond the reach of human reason.

Call this **Textbook School Mathematics (TSM)**. TSM has been the de facto national school curriculum for a long time.

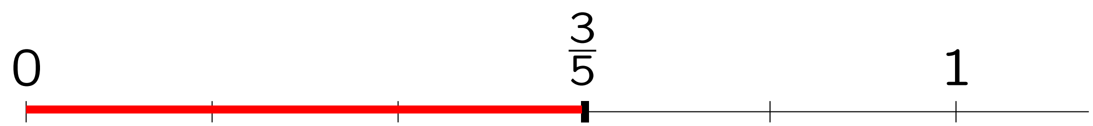
What may not be obvious is the fact that:

> Every topic in school mathematics *can* be made transparent and reasonable.

Let us go back to the previous examples and give a brief indication of how this can be done.

(1) $\dfrac{2}{7} \times \dfrac{3}{5} = ?$

We **define** a fraction such as $\frac{3}{5}$ as the length of a certain segment on the number line. Thus:



Then $\frac{2}{7} \times \frac{3}{5}$ is **defined** to be the total length of 2 parts when the segment of length $\frac{3}{5}$ is partitioned into 7 parts of equal length. We now explain why

$$\frac{2}{7} \times \frac{3}{5} = \frac{2 \times 3}{7 \times 5}$$

Here is the reason. How to divide a segment of length $\frac{3}{5}$ into 7 equal segments?

If we have to divide a segment of length $\frac{7}{5}$ into 7 equal segments, it is easy:

$$\frac{7}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}.$$

so each segment has length $\frac{1}{5}$.

If we write $\frac{3}{5}$ as $\frac{7\times 3}{7\times 5}$ (equivalent fractions), then again

$$\frac{7\times 3}{7\times 5} = \frac{3}{7\times 5}+\frac{3}{7\times 5}+\frac{3}{7\times 5}+\frac{3}{7\times 5}+\frac{3}{7\times 5}+\frac{3}{7\times 5}+\frac{3}{7\times 5}$$

So each part has length $\frac{3}{7\times 5}$. Two of them therefore have length $\frac{2\times 3}{7\times 5}$.

Thus, by definition of multiplication,

$$\frac{2}{7}\times\frac{3}{5} = \frac{2\times 3}{7\times 5}$$

(CCMS does this correctly in grades 4-5.)

(2) Solve: *Ann walks briskly and covers 3 miles the first hour. How many miles does she cover in 84 minutes?*

Here one has to explicitly assume that Ann walks **at a constant speed**. This concept of "constant speed" or "constant rate" requires very careful explanation.

Textbooks often give problems such as Ann's walk without mentioning constant speed.

Knowing that the preceding strategy only works for motions of constant speed, we now understand why the following cannot be done the same way:

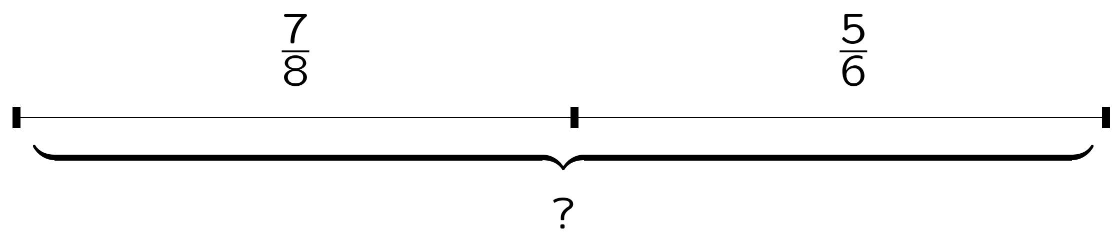
A stone is dropped from 144 ft. It drops 16 ft the first second. How much does it drop in 3 seconds?

Physics tells us that the stone does not fall at constant speed, so this is a different problem altogether.

(CCMS does *constant speed* somewhat better than the average in grade 6, though not as well as could be.)

(3) Adding fractions. $\mathbf{\frac{7}{8} + \frac{5}{6}}$

We are now combining the two segments of lengths $\frac{7}{8}$ and $\frac{5}{6}$, and want to find the total length.



Briefly: the first is 6×7 segments of length $\frac{1}{6 \times 8}$, and the second is 8×5 segments of length $\frac{1}{8 \times 6}$. So the total length is

$$(6 \times 7) + (8 \times 5) \text{ segments of length } \frac{1}{48}.$$

Thus the answer: $\frac{(6 \times 7)+(8 \times 5)}{48}$.

(CCMS does this correctly in grades 3-5.)

(4) What is a parabola?

A **parabola** is a curve that is similar (in the sense of "similar triangles") to the graph of x^2.

One then proves the fact that the graph of every quadratic function is similar the graph of x^2 (hence a parabola).

(CCMS mandates a correct definition of a parabola in High School Geometry.)

Pre-service teachers have to learn how to transform TSM into something transparent and reasonable in order to properly carry out their duties as classroom teachers, but so far, *institutions of higher learning have not done their job.*

Consequently, most of our teachers—who were taught TSM in K-12—are left with no choice but to teach their own students TSM when they go back to teach.

This is how TSM gets recycled from generation to generation.

CCMS is taking a first step to break down TSM and make school mathematics transparent and reasonable again.

CCMS is very possibly our last hope to break the vicious cycle of TSM for a long time to come.

Let us look ahead and ask **what next?**

The first requirement is **do no harm**.

An example of **doing harm**: in the 90's, LA Unified (LAUSD) once promulgated a Pacing Guide for teachers of Algebra 1 that required them to teach the quadratic formula in *May* and the concept of a square root in *June*.

K-12 mathematics education involves serious mathematics. Many instructional decisions, such as the LAUSD Pacing Guide, should take into account input from content experts.

In the Common Core era, involvement of very competent mathematicians is essential.

Why "very competent"?

A common misconception is that **any** mathematics professor is a content expert. Here is my thinking on the issue: I was given tenure at UC Berkeley in 1968, but I certainly would not have recommended anyone to consult me about K-12 math education back in 1968. I didn't know enough mathematics.

A deeper answer to “what next” will have to be professional development, professional development, and professional development (**PD**).

This is not the place to discuss pre-service PD (it must improve). A more pertinent question on this occasion is what you can do for your teachers in the district.

In order to implement CCMS, you will have to provide in-service PD that is **content-based and sustained over a long period of time.**

Most teachers need a replacement of their knowledge of TSM, because TSM is incompatible with CCMS and universities have not provided them with this replacement.

This kind of content-knowledge cannot be acquired in two or three fun-filled, half-day PD sessions each semester; it requires effort as well as sustained immersion in the mathematics.

PD means different things to different people at the moment.

To some, it means games, fun activities, new manipulatives, pedagogical strategies, and projects that you can directly bring back to your classroom.

To others, it means making teachers feel good about themselves, making them feel that that already know mathematics, and making them believe that mathematics can be learned without hard work.

The better kind of PD talks about children's mathematical thinking, skillful use of technology, teacher-student communication, and refined teaching practices.

While these are important issues for teaching, the kind of PD that is most urgently needed is the kind that provides content knowledge.

Most teachers need content knowledge about the basic mathematical topics of the school curriculum. They have to be able to teach these topics with precision, reasoning, and coherence, and in a way that is grade-level appropriate.

It is not high quality mathematics per se that they need to know, but **high-quality mathematics done from the vantage point of the school classroom.**

Effective PD must combine the best of both worlds.

This kind of PD cannot materialize without the contributions of all parties concerned: district supervisors, teachers, and university mathematicians.

For the teachers who need this kind of PD, they have to be willing to unlearn a lot of things picked up from TSM before they can pick up the requisite new knowledge. Will the school district encourage them to put in the sustained effort? Are they willing to put in the sustained effort?

Are district supervisors willing to do a serious re-thinking about PD: put content first, and find the funding for long-term PD?

There is a serious issue of finding professional developers who can provide such content knowledge.

Broadly speaking, only competent mathematicians possess this knowledge, but most of them know little about schools or are too involved in their own abstract world to be willing to do PD. The right peron *can* be found, but good administrative judgement will be critical.

Mathematicians can contribute in another way. They can serve as district consultants on hiring decisions.

Many professional developers claim to provide “content-based” PD. Informed administrative decisions that separate the wheat from the chaff will depend on getting good advice from mathematicians.

When all is said and done, the burden falls on the district supervisors. There is no substitute for great leadership.

The fate of CCMS is hanging in the balance: Can we get teachers who can makes sense of the mathematics they teach? Can we get teachers to teach mathematics in a way that is clear, precise, and supported by reasoning every step of the way?

Can we all contribute our share to make this happen?

Our children are waiting for an affirmative answer.