# Homework 8

due October 31, 2013

**1.** Let $f\colon X \to Y$ be an arbitrary mapping. Suppose $\mathscr{A} \subseteq \mathscr{P}(X)$ and $\mathscr{B} \subseteq \mathscr{P}(Y)$ to be families closed under finite intersection. For each of the following families

$$f_{**}\mathscr{A}, \qquad f^{**}\mathscr{A}, \qquad f^{*}{}_{*}\mathscr{B} \qquad \text{and} \qquad f_{*}{}^{*}\mathscr{B}$$

either prove that it has again the same property, or explain why it does not, in general (providing a correct counterexample will do).

Which of your answers would be different if $f$ was assumed to be surjective (respectively, injective)? Provide a correct reason in each case.

**2.** Do the same exercise for families closed under arbitrary unions.

**3.** Do the same exercise for topologies.

**4.** Do the same exercise for families directed below.

**5.** Do the same exercise for filter-bases.

**6.** Do the same exercise for upfamilies.

**7.** Do the same exercise for filters.