

## Early History of *Algebra*: a Sketch

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Algebra has its roots in the theory of quadratic equations which obtained its original and quite full development in ancient **Akkad** (Mesopotamia) at least 3800 years ago. In Antiquity, this earliest *Algebra* greatly influenced Greeks<sup>1</sup> and, later, **Hindus**. Its name, however, is of Arabic origin. It attests to the popularity in Europe of High Middle Ages of *Liber algebre et almuchabole* — the Latin translation of the short treatise on the subject of solving quadratic equations: **الْكِتَابُ الْمُخْتَصَرُ فِي حِسَابِ الْجَبْرِ وَالْمُقَابَلَةِ** *Al-kitābu 'l-muhtaṣaru fī ḥisābi 'l-ğabri wa-'l-muqābalati* (A summary of the calculus of *gebr* and *muqabala*).

The original was composed *circa* AD 830 in Arabic at the *House of Wisdom*—a kind of *academy* in Baghdad where in IX-th century a number of books were compiled or translated into Arabic chiefly from Greek and Syriac sources—by some **Al-Khwarizmi**<sup>2</sup> whose name simply means that he was a native of the ancient city of Khorezm (modern Uzbekistan).

Three Latin translations of his work are known: by Robert of Chester (executed in Segovia in 1140), by **Gherardo da Cremona**<sup>3</sup> (Toledo, *ca.* 1170) and by Guglielmo de Lunis (*ca.* 1250).

Al-Khwarizmi's name lives today in the word *algorithm* as a monument to the popularity of his other work, on **Indian Arithmetic**, which circulated in Europe in several Latin versions dating from before 1143, and spawned a number of so called *algorismus* treatises in XIII-th and XIV-th Centuries.

During the Middle Ages, *Algebra* was essentially limited to solving **polynomial equations** of degree  $\leq 3$ . The aforementioned treatise of Al-Khwarizmi deals only with quadratic

<sup>1</sup>The achievement of Greeks is immense; see the articles on **Eudoxus of Cnidos**, **Euclid**, **Archimedes**, **Nicomachus of Gerasa**, and especially **Diophantus**.

<sup>2</sup>مُوسَى الْخَوَارِزْمِيُّ مُحَمَّدُ بْنُ مُوسَى *Muḥammadu bnū Mūsā 'l-Ḥwārizmiyy*

<sup>3</sup>We owe to him the name for the *sine* function in Trigonometry.

equations. A native of Nishapur in Persia, **Omar Khayyam** (1048–1131), in the end of the XI-th Century employed geometrical methods to solving cubic equations. In Europe, Omar Khayyam is primarily known as the famous author of the collection of poems, *Rubayyat*. One of the earliest efforts to free this early *Algebra* from relying on geometric methods was due to **Al-Karaji**<sup>4</sup> (953–ca. 1029), whose family seems to have come to Baghdad from the Persian city of Karaj.

A major breakthrough occurred in 1515 when a native of Bologna, **Scipione dal Ferro** (1465–1526), solved the cubic equation algebraically. Another Bolognese, **Lodovico Ferrari** (1522–1565), in 1540 discovered the solution of the quartic equation.<sup>5</sup>

Apart from isolated attempts, like the use of letters to denote numbers by **Jordanus de Nemore** (1225–1260), or the introduction of symbols + and – by **Johannes Widman** (1462–1498) in 1489, no systematic “algebraic” notation was used by early algebraists before the XVI-th Century. Instead of symbols and equations they have been using descriptive sentences. What we call today “algebraic” notation makes its entry into Mathematics in the treatise *In artem analyticam isagoge* published in 1591 in Tours by **François Viète** (1540–1603), a native of Vandée (Western France).

Modern Algebra commences with the publication in 1830 of *Treatise on Algebra* by **George Peacock** (1791–1858). Within the next hundred years Algebra becomes a theory of mathematical *structures*.

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<sup>4</sup>أَبُو بَكْرِ بْنِ مُحَمَّدِ بْنِ الْحُسَيْنِ الْكَرْجِي *Ābū Bakri bnu Muḥammadi bni 'l-Ḥusayni 'l-Karaḡi*

<sup>5</sup>The history of Italian mathematical discoveries in the first half of XVI-th Century reads like a thriller; see the articles on **Pacioli**, **dal Ferro**, **Tartaglia**, **Cardano**, **Ferrari**, and **Bombelli**.