Homotopic paths Let E be a subset of $\mathbb{R}^m$. Suppose we are given two paths in E, $\gamma_\mathbf{0}\colon [a,b] \to E$ and $\gamma_\mathbf{1}\colon [a,b] \to E$, having the same starting point

$$\gamma_\mathbf{0}(a) = \gamma_\mathbf{1}(a) = \mathbf{a}$$

and the same endpoint

$$\gamma_\mathbf{0}(b) = \gamma_\mathbf{1}(b) = \mathbf{b}\ .$$

We say that these paths are **homotopic** (**in set** E) if there exists a contin-

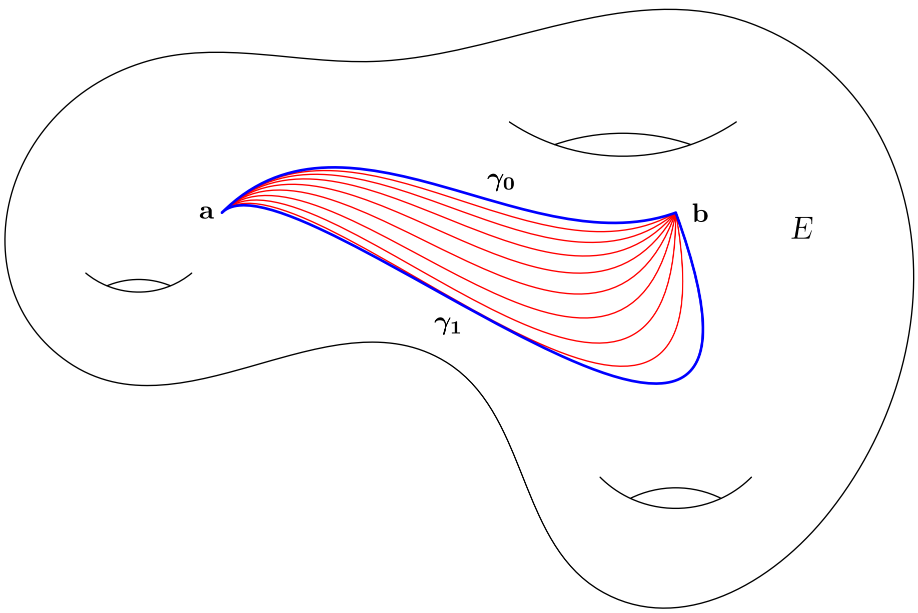

Figure 1: An example of a homotopy between paths.

uous function $H\colon \mathcal{I} \to E$ defined on the rectangle

$$\mathcal{I} := [a,b] \times [0,1] = \left\{ \begin{pmatrix} t \\ u \end{pmatrix} \in \mathbb{R}^2 \,\middle|\, a \le t \le b\ ,\ 0 \le u \le 1 \right\} \tag{1}$$

such that

$$H\left(\begin{pmatrix} t \\ 0 \end{pmatrix}\right) = \gamma_\mathbf{0}(t) \;\;\&\;\; H\left(\begin{pmatrix} t \\ 1 \end{pmatrix}\right) = \gamma_\mathbf{1}(t) \tag{2}$$

for *all* $t \in [a, b]$, and

$$H\left(\begin{pmatrix} a \\ u \end{pmatrix}\right) = \mathbf{a} \qquad \& \qquad H\left(\begin{pmatrix} b \\ u \end{pmatrix}\right) = \mathbf{b} \tag{3}$$

for *all* $u \in [0, 1]$. Function H in this case is called a **homotopy** *between*

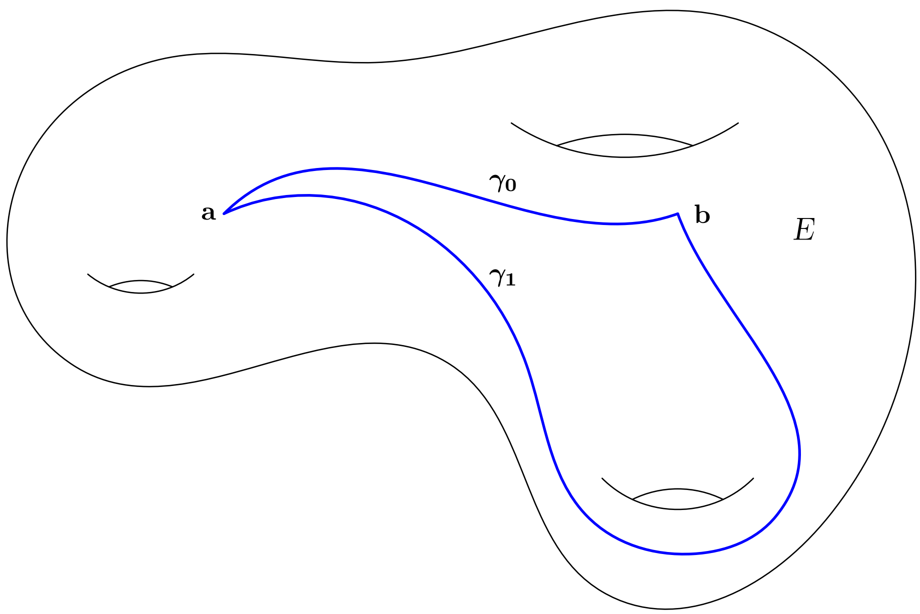

Figure 2: An example of *nonhomotopic* paths: there is no homotopy between these two path in set E.

$\boldsymbol{\gamma_0}$ and $\boldsymbol{\gamma_1}$. If the paths are differentiable and homotopic, then one can always find a differentiable homotopy.