Early History of *Algebra*: a Sketch

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Algebra has its roots in the theory of quadratic equations which obtained its original and quite full development in ancient Akkad (Mesopotamia) at least 3800 years ago. In Antiquity, this earliest *Algebra* greatly influenced Greeks[1] and, later, Hindus. Its name, however, is of Arabic origin. It attests to the popularity in Europe of High Middle Ages of *Liber algebre et almuchabole* — the Latin translation of the short treatise on the subject of solving quadratic equations: اَلْكِتَابُ اَلْمُخْتَصَرُ فِي حِسَابِ اَلْجَبْرِ وَاَلْمُقَابَلَةِ *Al-kitābu 'l-muḫtaṣaru fī ḥisābi 'l-ğabri wa-'l-muqābalati* (A summary of the calculus of *gebr* and *muqabala*).

The original was composed *circa* AD 830 in Arabic at the *House of Wisdom*—a kind of *academy* in Baghdad where in IX-th century a number of books were compiled or translated into Arabic chiefly from Greek and Syriac sources—by some Al-Khwarizmi[2] whose name simply means that he was a native of the ancient city of Khorezm (modern Uzbekistan).

Three Latin translations of his work are known: by Robert of Chester (executed in Segovia in 1140), by Gherardo da Cremona[3] (Toledo, *ca.* 1170) and by Guglielmo de Lunis (*ca.* 1250).

Al-Khwarizmi's name lives today in the word *algorithm* as a monument to the popularity of his other work, on Indian Arithmetic, which circulated in Europe in several Latin versions dating from before 1143, and spawned a number of so called *algorismus* treatises in XIII-th and XIV-th Centuries.

During the Middle Ages, *Algebra* was essentially limited to solving polynomial equations of degree $\leqslant 3$. The aforementioned treatise of Al-Khwarizmi deals only with quadratic equations. A native of Nishapur in Persia, Omar Khayyam (1048–1131), in the end of the XI-th Century employed geometrical methods to solving cubic equations. In Europe, Omar Khayyam is primarily known as the famous author of the collection of poems, *Rubayyat*. One of the earliest efforts to

[1] The achievement of Greeks is immense; see the articles on Eudoxus of Cnidos, Euclid, Archimedes, Nicomachus of Gerasa, and especially Diophantus.

[2] مُحَمَّدُ بْنُ مُوسَى اَلْخْوَارِزْمِيّ *Muḥammadu bnu Mūsā 'l-Ḫwārizmiyy*

[3] We owe to him the name for the *sine* function in Trigonometry.

free this early *Algebra* from relying on geometric methods was due to Al-Karaji[4] (953–*ca.* 1029), whose family seems to have come to Baghdad from the Persian city of Karaj.

A major breakthrough occurred in 1515 when a native of Bologna, Scipione dal Ferro (1465–1526), solved the cubic equation algebraically. Another Bolognese, Lodovico Ferrari (1522–1565), in 1540 discovered the solution of the quartic equation.[5]

Apart from isolated attempts, like the use of letters to denote numbers by Jordanus de Nemore (1225–1260), or the introduction of symbols $+$ and $-$ by Johannes Widman (1462–1498) in 1489, no systematic "algebraic" notation was used by early algebraists before the XVI-th Century. Instead of symbols and equations they have been using descriptive sentences. What we call today "algebraic" notation makes its entry into Mathematics in the treatise *In artem analyticam isagoge* published in 1591 in Tours by François Viète (1540–1603), a native of Vandèe (Western France).

Modern Algebra commences with the publication in 1830 of *Treatise on Algebra* by George Peacock (1791–1858). Within the next hundred years Algebra becomes a theory of mathematical *structures*.

[4] أَبُو بَكْرِ بْنُ مُحَمَّدِ بْنِ آلْحُسَيْنِ آلْكَرَجِ *Ābū Bakri bnu Muḥammadi bni 'l-Ḥusayni 'l-Karaği*

[5] The history of Italian mathematical discoveries in the first half of XVI-th Century reads like a thriller; see the articles on Pacioli, dal Ferro, Tartaglia, Cardano, Ferrari, and Bombelli.