Solutions to 2nd in-class Quiz

Will Johnson

February 16, 2015

1. Integrate
$$\int \frac{dx}{1+\sqrt{x}}.$$

   Solution. Do a u-substitution with $u = 1+\sqrt{x}$. So $x = (u-1)^2$, and $dx = 2(u-1)\,du$. Thus
$$\int \frac{dx}{1+\sqrt{x}} = \int \frac{2(u-1)\,du}{u} = 2\int (1-1/u)\,du = 2u-2\ln u+C = 2(1+\sqrt{x})-2\ln(1+\sqrt{x})+C.$$

   □

2. Break the following into partial fractions:

   (a) $1/(x^2-x)$

   Solution. This rational function is already a proper fraction, so we don't have to do any long division. The denominator factors as $(x-1)x$, so the partial fraction decomposition will be of the form
$$\frac{1}{x^2-x} = \frac{A}{x-1} + \frac{B}{x}$$
   for some constants A and B. Clearing denominators, this should be true:
$$1 = A\cdot x + B\cdot(x-1).$$
   Setting $x = 1$, we get
$$1 = A\cdot 1 + B\cdot 0,$$
   so $A = 1$. Setting $x = 0$ instead, we get
$$1 = A\cdot 0 + B\cdot(0-1) = -B,$$
   so $B = -1$. Thus, the solution is
$$\frac{1}{x^2-x} = \frac{1}{x-1} + \frac{-1}{x}$$

   □

(b) $x^3/(x+1)$

Solution. To start, we do polynomial long division, which is hard to typeset. But essentially what happens is the following:

$$\begin{aligned}\frac{x^3}{x+1} &= \frac{x^3+x^2}{x+1} - \frac{x^2}{x+1} = x^2 - \frac{x^2}{x+1} \\ &= x^2 - \frac{x^2+x}{x+1} + \frac{x}{x+1} = x^2 - x + \frac{x}{x+1} \\ &= x^2 - x + \frac{x+1}{x+1} - \frac{1}{x+1} = x^2 - x + 1 - \frac{1}{x+1}.\end{aligned}$$

Now we need to break $\frac{1}{x+1}$ into partial fractions, except it is already broken so. The solution is

$$\frac{x^3}{x+1} = x^2 - x + 1 - \frac{1}{x+1}.$$

□

3. Which are true and which are false?

(a) For every x, $x \cdot 0 = 0$.

(b) For every x, $x \cdot 1 = 1$.

(c) There is an x such that $x^2 = 2$.

(d) For every x, there is a y such that $y > x$.

Solution. The first statement is true (it's the statement that if you multiply any number by zero, the result is zero). The second statement says that if you multiply any number by 1, the result is 1. This is false. For example, 23 times 1 is not 1.

The third statement is true—the square root of 2 is one such x.

The fourth statement says that given any number x, we can find some other number which is even bigger than x. This is true. For example, $x + 1$ is bigger than x. □