Topics to know for the second Midterm

October 28, 2014

We don't have any idea what will be on the second midterm, but here's my best guess about what sort of things would be good to know:

- Exponent and logarithm laws.
- The identities
$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
as well as the double angle formulas
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x) = 1 - 2\sin^2(x) = 2\cos^2(x) - 1$$
- All the limit laws, including the fact that limits pass through continuous functions. Know the exact assumptions of the various limit laws.
- Know the derivatives of all the trig and inverse trig functions, as well as the derivatives of a^x and $\log_a(x)$.
- Be able to use the chain rule to take derivatives of complicated functions.
- Know the proofs that the derivative of sine is cosine, the derivative of cosine is negative sine, the derivative of natural logarithm is $1/x$, and that the derivative of the exponential function is itself.
- Know the exact statements of the chain rule, product/sum/quotient rules, the extreme value theorem, the intermediate value theorem, the mean value theorem (including Rolle's theorem), and Fermat's theorem. Know what assumptions have to be made, and know counterexamples when the assumptions fail.
- Know the proofs of the chain rule, the product rule, Fermat's theorem, Rolle's theorem, and maybe the mean value theorem. Know why the naive proof of the chain rule doesn't work.

- Know that differentiable functions are continuous, including the more specific statement that if f is differentiable at a, then it's continuous at a.
- Know how to verify that

  $$f(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

  is differentiable.
- Know the definition of limit, continuity, and differentiability.
- Know how to do a problem like: find all tangent lines of the circle $x^2 + y^2 = 1$ which pass through the point $(5, 1)$.
- Know how to calculate $\lim_{x \to 0} f(x)/x$ when $f(x)$ is a differentiable function and $f(0) = 0$, by writing this as a derivative. Similarly, know how to calculate things like $\lim_{x \to 1}(x^{1000} - 1)/(x - 1)$ by rewriting this limit as a derivative.
- Know what assumptions have to be made to do implicit differentiation. (See the top of page 210 in the textbook.)
- Know the following specific limits

  $$\lim_{x \to 0} \frac{\sin(x)}{x} = 1, \qquad \lim_{xto0} \frac{1 - \cos x}{x} = 0,$$

  $$\lim_{x \to 0}(1 + x)^{1/x} = e, \qquad \lim_{n \to \infty}(1 + 1/n)^n = e, \qquad \lim_{x \to 0} \frac{\ln(1 + x)}{x} = 1.$$

  Know why the latter three are true, or at least why they're all nearly equivalent to each other.
- Know how to check differentiability, and find the derivative, of piecewise functions like

  $$f(x) = \begin{cases} 1 + x & x \geq 0 \\ e^x & x < 0 \end{cases}$$

  or the one in problem 2.8.56 in the book.
- Know how to do a change of variables in a limit.
- Know how to use the intermediate value theorem to see that a function like $x^3 - 2x^2$ or $x + e^x$ has as its range all of $\mathbb{R}$.
- Know how to calculate $\lim_{x \to 0} f(x)/g(x)$ in the case where $f(x)$ and $g(x)$ are differentiable functions, and $f(0) = g(0) = 0 \neq g'(0)$:

  $$\lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{f(x)/x}{g(x)/x} = \frac{\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}}{\lim_{x \to 0} \frac{g(x) - g(0)}{x - 0}} = \frac{f'(0)}{g'(0)}.$$