(1) Mark each of the following questions true (T) or false (F). Provide a sentence or two justifying each answer.

(a) If $x \equiv y \pmod{m}$ then $ax \equiv ay \pmod{m}$.

(b) If $ax \equiv ay \pmod{m}$ then $x \equiv y \pmod{m}$.

(c) The function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = \lfloor \frac{x}{2} \rfloor$ is surjective.

(d) The set of integers and the set of even integers have the same cardinality.

(e) The positive real numbers are countable.

(f) Let $\mathbb{R}$ be the domain, and let $P(x, y)$ be the statement $y^2 = x$. Determine the truth value of the following statement:
   $\forall x \exists y \ P(x, y)$.
(2) Prove that if $m$ is a positive integer of the form $4k + 3$ for some non-negative integer $k$, then $m$ is not the sum of the squares of two integers.
(3) Prove that $A \cup B = \overline{A} \cap \overline{B}$. 
(4) Computation.
   • Write the number 466 in base 9.

   • What is the sum of the first $n$ entries of the sequence 1, 3, 9, 27, 81, \ldots ?

   • Calculate $(47^{100} + 25^4) \mod 23$.

   • What is the contrapositive of the statement “If my cell phone rings, I disturb the lecture.”
(5) Prove that if $p$ is prime, the only solutions of $x^2 \equiv 1 \pmod{p}$ are integers $x$ such that $x \equiv 1 \pmod{p}$ or $x \equiv -1 \pmod{p}$. 
(6) Show that if $a$ and $b$ are positive integers, then $ab = \gcd(a, b) \cdot \lcm(a, b)$.