What's a good way to represent a graph?
Listing all vertices & edges is cumbersome.

**Def:** Let $G = (V, E)$ be an undirected graph w/ $|V| = n$. Denote vertices by $V_1, \ldots, V_n$. The adjacency matrix $A$.

**Ex:**

**Ex:**

**Ex:** Draw a graph w/ adj matrix.

**Obs:** An adj matrix of undirected graph.
**Def:** If \( G = (V, E) \) is a directed graph, its adj. matrix \( A \) (an \( n \times n \) matrix) is the \( n \times n \) matrix s.t.

\[
A = \begin{pmatrix}
0 & \cdots & 1 & \cdots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \cdots & 1 \\
\end{pmatrix}
\]

**Ex:**

Not in general a

Another way to represent graph:

**Def:** Let \( G = (V, E) \) be undirected graph.
Let \( v_1, \ldots, v_n \) be vertices and \( e_1, \ldots, e_m \) be edges.
The incidence matrix

\[
I = \begin{pmatrix}
1 & 1 & \cdots & \cdots & 1 \\
1 & 1 & \cdots & \cdots & 1 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
1 & 1 & \cdots & \cdots & 1 \\
\end{pmatrix}
\]

**Ex:**

What does it mean for two graphs to be the "same"?

**Def:** The simple graphs \( G_1 = (V_1, E_1) \) and \( G_2 = (V_2, E_2) \) are isomorphic if
Ex: Are these 2 graphs isomorphic?

Note: If $G_1$ and $G_2$ are isomorphic, they must have

If $G_1$ and $G_2$ are 2 graphs w/ n vertices, can be hard to determine whether they are isomorphic:

If we think 2 graphs not isomorphic, good strategy is to
**Def:** A property preserved by isomorphism is 

\[ \text{a} \]

**Ex:** If \( G_1 \) and \( G_2 \) are isomorphic and \( G_1 \) has \( n \) vertices,

**Ex:** Show that these 2 graphs are not iso.
§ 10.4 Connectivity

A path in graph is

Def: Let \( n \in \mathbb{N} \) and \( G \) an undirected graph. A path of length \( n \) from \( u \) to \( v \) is

If graph simple, can just give

Def: Path is circuit if

Def: Let \( n \in \mathbb{N} \) and \( G \) a directed graph. A path of length \( n \) from \( u \) to \( v \) is
Ex of paths in graphs from real life.

Ex: Let $G = (V, E)$ where $V$ = set of places, $E =$

Def: An undirected graph called connected if

Ex: In previous example, if $V =$

Ex: Which of these graphs is connected?

Recall: A subgraph of $G = (V, E)$ is a graph
Def: A connected component of a graph $G$ is

Ex: What are connected components of $G = \text{...}$

For directed graphs, 2 notions of connected:

Def: A directed graph is strongly connected.

Def: A directed graph is weakly connected.

Ex: Is $G$ st. conn? weak conn? $G = \text{...}$
Paths/circuits can be helpful in determining questions of isomorphism.

Ex: if \( f: G_1 \to G_2 \) is a graph isomorphism, and

Are these 2 graphs iso?