Recall: A relation $R$ on set $A$ is an equivalence relation if it is:

* Reflexive
  
* Symmetric
  
* Transitive

Ex: Let $S$ be relation on $\mathbb{R}$ defined by $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} | x + y = 0 \}$.

Show that $S$ is equivalence relation.

Reflex: Need to show $x + x = 0 \quad \forall x \in \mathbb{R}$

Symm: Suppose $(x, y) \in S$. Need to show $(y, x) \in S$.

Trans: Suppose $(x, y) \in S$ and $(y, z) \in S$. Need to show $(x, z) \in S$. 
Def: An (undirected) graph \( G = (V, E) \) is

Def: If there are several edges between same 2 endpoints, called

Def: A loop is

Ex: Let \( V = \) set of all students in class.
Let \( E = \)

Part of graph might look like:

For some types of information, a directed graph is better.

Def: A directed graph or digraph \( (V, E) \) is
Ex: Let $V$ = set of all species.

Draw edge $u$ to $v$ whenever

Ex: The web (internet). Let $V$ =

Q: What if we want to model

Q:

\[ f(10, 2) \]

Def: Two vertices $u$ and $v$ in undirected graph $G$ are adjacent in $G$ if
Def: The degree of a vertex in an undirected graph is the number of edges incident to it.

Handshaking Theorem: Let $G = (V, E)$ be a graph with $e$ edges. Then

Pf: 

Ex: 
"Handshaking" because:

Theorem:

PF:

Special kinds of graphs:
The complete graph $K_n$

The cycle $C_n$ has
The wheel $W_n$

The $n$-cube $Q_n$

**Def:** A graph $G = (V, E)$ is bipartite if $V$ can

**Ex:**
Thm: A simple graph is bipartite iff

\[ \text{Def: Let } G = (V, E) \text{ be an undirected graph w/ } |V| = n. \]
Denote vertices by \( v_1, \ldots, v_n \). The adjacency matrix \( A \) (or \( A_c \)) of \( G \) is
\varepsilon_x^* \\
\varepsilon_x^* \\
Obs.