Def: Let \( S \) be sample space of an experiment w/ a finite or countable number of outcomes. A probability distribution \( \mathbf{p} \) is a function \( \mathbf{p} : S \rightarrow \mathbb{R} \) s.t.:

(i.) 
(ii.)

Call \( \mathbf{p}(s) \) the

Def: Let \( S \) be set w/n elements. The uniform distribution

Def: If \( E \subseteq S \) is an event, the probability of \( E \) is

\[
\text{Theorem:} \quad (1) \quad p(\overline{E}) = 1 - p(E) \\
(2) \quad p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2) \\
(3) \quad \text{If } E_1, E_2, \ldots \text{ is a sequence of pairwise disjoint events in sample space } S, \text{ then } \quad p(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} p(E_i)
\]

Conditional Probability

Suppose we flip a fair coin 3 times. Let \( F \) be event that first flip is T. Let \( E \) be event that an odd number of T appears.
Suppose we know that F occurs. Given this knowledge, what is $p(E)$?

\[ \text{Sol:} \]

To find conditional prob. of $E$ given $F$, use

**Def:** Let $E$ and $F$ be events w/ $p(F) > 0$. The **conditional probability** of $E$ given $F$, denoted $p(E|F)$, is defined as $p(E|F) = \ldots$

**Ex:** What is the conditional probability that a family w/ 4 children has 4 girls, given they have at least 1 girl?

\[ \text{Sol:} \]
Independence: Suppose a coin is flipped 3 times. Does knowing that the first flip is tails (F) alter the probability that tails come up an odd number of times (event E)?

Note: \[ p(E|F) = \frac{p(E \cap F)}{p(F)} \] so if \( p(E|F) = p(E) \), then

Def: The events E and F are independent iff

Ex: Assume that each of the 4 ways a family can have 2 kids is equally likely.
Let E - event that a family with 2 kids has 2 boys
Let F - event that " " " " has at least one boy.
Are E and F independent?

Sol:
Bernoulli trials

Def: Suppose an experiment has 2 possible outcomes (e.g., coin flip). Each performance of such an experiment

Ex: Have biased coin s.t. \( p(H) = \frac{1}{5} \). What is prob that exactly 2 heads come up, when coin is flipped 3 times, assuming flips are indep of each other?

Sol:

Theorem: The prob. of exactly \( k \) successes in \( n \) indep Bernoulli trials, w/ prob. \( p \) of success, & prob. \( q = 1-p \) of failure, is
Random Variable

**Def**: A random variable is a function from the set of outcomes to the set of real numbers.

**Note**: Random variable is a FUNCTION, not a variable.

**Ex**: Suppose coin flipped 3 times. Let $X(t)$ be random var that equals number of heads that appear when it is the outcome.

\[
X(\text{HHH}) = \\
X(\text{HHT}) = X(\text{HTH}) = X(\text{THH}) = \\
X(\text{HTT}) = X(\text{THT}) = X(\text{TTH}) = \\
X(\text{TTT}) =
\]

§7.3 Bayes' Theorem

Sometimes we want to assess the probability that a particular event occurs on the basis of

**Ex**: We have two boxes $B_1, B_2$. $B_1$ contains two green balls & seven red balls; $B_2$ has four green balls & three red balls. Bob selects a ball by first: choosing one of two boxes at random, then selecting one of the balls in the box at random. If Bob has selected a red ball, what is the prob that he selected ball from $B_1$?
Bayes' Theorem: Suppose that $E$ and $F$ are events from a sample space $S$ s.t. $p(E) \neq 0$ and $p(F) \neq 0$. Then

$$p(F|E) =$$

Ex: Suppose one person in 100,000 has very rare disease for which there is fairly accurate test. Test is correct 99% of time when given to someone w/ disease & is correct 99.5% of time when given to someone who doesn't have disease. Find:

• the prob that someone who tests pos. for disease has disease

Sol: