Review #1

Ex: In 6-card poker, you got dealt 6 cards from a deck of 52.
[Recall: deck has 4 suits, 13 kinds A23...K]

(a) What is the probability you got dealt a hand with a four-of-a-kind and also a two-of-a-kind (e.g. 4 queens & two 5's)

\[ \frac{13 \cdot \binom{4}{4} \cdot 12 \cdot \binom{4}{2} \text{ choose 2nd kind}}{\binom{52}{6}} \]

(b) Prob you got dealt a hand w/ two three-of-a-kinds?

\[ \frac{\binom{13}{2} \cdot \binom{4}{3} \cdot \binom{4}{3}}{\binom{52}{6}} \]

(c) What is the prob. you do not get at least two cards of the same kind?

need to use 6 different kinds & choose one card of each kind

\[ \frac{\binom{13}{6} \cdot 4^6}{\binom{52}{6}} \]
Ex: Find the inverse of 19 mod 102.

Invert the Euclidean algorithm...

\[102 = 5 \cdot 19 + 7\]
\[19 = 2 \cdot 7 + 5\]
\[7 = 1 \cdot 5 + 2\]
\[5 = 2 \cdot 2 + 1\]

So \[1 = 5 - 2 \cdot 2\]
\[= 5 - 2 \cdot (7 - 1 \cdot 5)\]
\[= 3 \cdot 5 - 2 \cdot 7\]
\[= 3 \cdot (19 - 2 \cdot 7) - 2 \cdot 7\]
\[= 3 \cdot 19 - 8 \cdot 7\]
\[= 3 \cdot 19 - 8 \cdot (102 - 5 \cdot 19)\]
\[= 43 \cdot 19 - 8 \cdot 102\]

\[1 = 43 \cdot 19 - 8 \cdot 102 \quad \Rightarrow \quad 1 = 43 \cdot 19 \quad (\text{mod } 102)\]

\[\therefore \text{ the inverse is } 43.\]

Check that \[43 \cdot 19 - 1\] is divisible by 102:
\[43 \cdot 19 - 1 = 816 = 102 \cdot 8\]
Ex: Find the inverse of \( 4 \mod 5 \).

Here it's easier to just guess & check.
Need to find \( x \) s.t. \( 4x \equiv 1 \mod 5 \)

\[
\begin{align*}
4 \cdot 1 &= 4 \equiv 4 \mod 5 \\
4 \cdot 2 &= 8 \equiv 3 \mod 5 \\
4 \cdot 3 &= 12 \equiv 2 \mod 5 \\
4 \cdot 4 &= 16 \equiv 1 \mod 5 & \therefore 4 \text{ is the inverse of } 4 \mod 5
\end{align*}
\]

Ex: Find the inverse of \( 2 \mod 5 \)

\[
\begin{align*}
2 \cdot 1 &= 2 \equiv 2 \mod 5 \\
2 \cdot 2 &= 4 \equiv 4 \mod 5 \\
2 \cdot 3 &= 6 \equiv 1 \mod 5 & \therefore 3 \text{ is the inverse of } 2 \mod 5
\end{align*}
\]
Ex: Suppose that 8% of all bicycle racers use steroids, that a bicyclist who uses steroids tests positive for steroids 96% of the time, and a bicyclist who doesn't use steroids tests positive for steroids 9% of the time. What is the probability that a randomly selected bicyclist who tests positive for steroids actually uses steroids?

Bayes' Theorem: Suppose that $E$ and $F$ are events from sample space $S$ such that $p(E) \neq 0$ and $p(F) \neq 0$. Then

$$p(F \mid E) = \frac{p(E \mid F) p(F)}{p(E \mid F) p(F) + p(E \mid \overline{F}) p(\overline{F})}$$

What should we define $F$ and $E$ to be? Look at what question is being asked?

$F =$ event that bicyclist uses steroids.

$E =$ “tests positive for steroids”.

Now let's identify the probabilities we've given:

$8\% = 0.08 \Rightarrow p(F)$

$96\% = 0.96 \Rightarrow p(E \mid F)$

$9\% = 0.09 \Rightarrow p(E \mid \overline{F})$

To use them, also need to know $p(\overline{F}) = ?$ 0.92
\[ p(F|E) = \frac{p(E|F)p(F)}{p(E|F)p(F) + p(E|\bar{F})p(\bar{F})} \]

\[ = \frac{0.96 \cdot 0.08}{0.96 \cdot 0.08 + 0.09 \cdot 0.92} \approx 0.481 \]

\[ \approx 48.1\% \]

What if we forgot Bayes' Theorem?

We still start by defining \( E \) and \( F \) as above, noticing that \( p(F) = 0.08 \)

and that we \( p(E|F) = 0.96 \) \( \oplus \)

want to find \( p(E|\bar{F}) = 0.09 \)

\( p(F|E) \).

By def, \( p(F|E) = \frac{p(E \cap F)}{p(E)} \) \( \oplus \) need to find

What can we deduce from \( \oplus \)?

By def, \( p(E|F) = \frac{p(E \cap F)}{p(F)} \).

Since we know these numerators \( \uparrow \), can solve for \( p(E \cap F) \).

Since we know \( p(F) \), can get \( p(\bar{F}) \) \( (= 1 - p(F)) \).

Also \( p(E|\bar{F}) = \frac{p(E \cap \bar{F})}{p(\bar{F})} \)

We know \( \uparrow \) and \( \uparrow \) so can compute \( p(E \cap \bar{F}) \).
Finally to compute $p(\varepsilon)$, use fact that

$$p(\varepsilon) = p(\varepsilon \cap F) + p(\varepsilon \cap \bar{F}).$$

Now can compute $\oplus$. 