Universal instantiation:
Given the premise $\forall x P(x)$, we can conclude that

Ex:

<table>
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<tr>
<th>Rule of Inference</th>
<th>Name</th>
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Ex: Show that premises
Let $S_n(x)$ denote
let $A_n(x)$ denote
The domain is

1. Step

2.

3.

4.

5.

§1.7 Introduction to Proofs

A proof is a valid argument that establishes the truth of a mathematical argument.

A theorem is
A proposition is
A lemma is
A corollary is
A conjecture is
1. **Direct proof** of $p \Rightarrow q$.

**Rk:** To prove theorem of form $\forall x \ (P(x) \Rightarrow Q(x))$, our goal is to

**Def:** The integer $n$ is **even** if there

$n$ is **odd** if

**Ex:** Show that the sum of two odd integers is even.

**Proof:**
2. **Proof by contraposition:**

Ex: Let \( n \in \mathbb{Z} \) (the integers). Show that if \( n^2 \) is even then \( n \) is even.

Proof:

**Note:** Direct proof is harder here...

3. **Proof by contradiction:**

Def: A real number \( r \) is **rational** if
Ex: Prove that $\sqrt{2}$ is irrational via $\uparrow$.

Proof:

Ex: Show that at least 10 of any 64 days chosen must fall on the same day of the week.

pf: $\uparrow$
Proofs of equivalence:
To prove a theorem that is a biconditional statement, i.e., \( p \iff q \), we show:

**Rt:** iff shorthand for “if and only if”

**Ex:**

**Pf:**

Proof by Case: Sometimes it is helpful to divide up the proof into cases...

**Ex:**

**Ex:**

Q: What cases should we consider?
Without loss of generality (WLOG):

Ex:
Pf:
Open Problems:

Example: