Math 55  Lecture 25  8/10.5

Def: An Euler circuit in graph $G$ is

$\exists \; \Gamma = \Gamma$

Make graph:

Question becomes:

$\Gamma =$

Note: 

$(1) \quad \text{If } v \text{ is the start/end vertex, then it must have}$

$(2) \quad \text{If } v \text{ is not the start/end vertex, then for}$
Note: We always assume graphs have

**Theorem:** A connected graph \( G \) with at least 2 vertices has an Euler circuit iff

**Example:** Do these graphs have Eulerian circuit?

**Lemma:** Any multigraph with at least 2 vertices, s.t. all degrees are even, has

**Proof:**
Proof of Theorem: We already proved that if $G$ has an $E.$ circuit, we need to show:

**Step 1:** Use Lemma to construct

**Step 2:**

**Step 3:** Remove edges of $C_1$ from $H_1$, get subgraph $H_2$. 

Ex: Does $G$ have an Eulermian circuit?

Step 1: Find a simple circuit $C_0$.

Step 2: Let $H_1$ be the graph of unused edges of $G$.

$$G = \begin{align*}
\begin{tikzpicture}
\node (a) at (0,0) {}; \\
\node (b) at (2,0) {}; \\
\node (c) at (1,1) {}; \\
\node (d) at (1,-1) {}; \\
\node (e) at (-1,0) {}; \\
\draw (a) -- (b) -- (c) -- (a); \\
\draw (a) -- (d) -- (c); \\
\draw (b) -- (e) -- (c); \\
\draw (b) -- (d); \\
\end{tikzpicture}
\end{align*}$$
Step 3: Let $H_2$ be graph of unused edges.
Related concept: Euler path

**Def:** An Euler path in G is

**Thm:** A connected graph has an Euler path but not an Euler circuit if and only if

**Ex:**

**Note:**