Math 55 Lecture 22  §10.1, p. 2

1st review equiv. rel’n...

**Recall:** A relation $R$ on set $A$ is subset of $A \times A$. It is an equiv. rel’n if it is:

- Reflexive
- Symmetric
- Transitive

**Ex:** Let $S$ be relation on $\mathbb{R}$ defined by:

$$S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x = y\}$$

Show that $S$ is equivalence relation.

**Reflex:** Need to show

**Symm:** Suppose $(x, y) \in S$. Need to show

**Trans:** Suppose $(x, y) \in S$ and $(y, z) \in S$. Need to show
Def: An (undirected) graph $G = (V, E)$ is

Def: If there are several edges between the same 2 endpoints, called

Def: A loop is

Can use graphs to represent information.

Ex: Let $V =$ set of all students in class.

Let $E =$

Part of graph might look like:

For some types of information, a

Def: A directed graph or digraph $(V, E)$ is
Ex: Let $V =$ set of all species. Draw edge $u$ to $v$ whenever

Ex: The web (internet). Let $V =$ set of all websites. Draw edge $u$ to $v$ if

Q: What if we want to model people

Q: Same question but people $u$ / $v$

$\phi_{10.2}$

Def: Two vertices $u$ and $v$ in undirected graph $G$ are
Def: The degree of vertex in undirected graph is

What do we get if we add all degrees of

Handshaking Theorem: Let $G = (V, E)$ be graph with edges. Then

Pf:
“Handshaking” because:

**Theorem:** A graph has an even number of vertices if

**PF:**

Special kinds of graphs:

The complete graph $K_n$

The cycle $C_n$ has
The wheel $W_n$ is

The $n$-cube $Q_n$

**Def:** A graph $G=(V,E)$ is bipartite if $V$ can

**Ex:**
Thm: A simple graph is bipartite if and only if

§10.3

What's a good way to represent a graph? Listing all vertices and edges is cumbersome.

Def: Let $G = (V, E)$ be an undirected graph with $|V| = n$. Denote vertices by $V_1, \ldots, V_n$. The adjacency matrix

Ex:
Ex: Draw a graph with adjacency matrix \[
\begin{pmatrix}
0 & 0 & 0 \\
0 & 1 & 1 \\
0 & 1 & 0 \\
\end{pmatrix}
\]

Obs: An adjacency matrix of an undirected graph is symmetric, i.e.