Math 55 Lecture 2: Predicate + Quantifiers

Let $P(x)$ be the statement $x+2=10$.
Recall: Not a proposition, because called a propositional function.
Once we assign value to $x$,

Can also have statements involving several variables.
Ex: Let $Q(x,y)$ denote statement $x=y+3$.

Another way to create a proposition from propositional function involves quantifiers...
English words used in quantifiers include...

Def: The domain of a propositional function $P(x)$

Def: We use notation $\forall x \ P(x)$ to denote the statement

Ex: Let $P(x)$ be the statement "$x^2 < x$"
What is truth value of $\forall x \ P(x)$, where
the domain consists
How about if domain
"A" is the universal quantifier.

**Def:** We use notation $\exists x \ P(x)$ to denote the statement

Many other quantifiers besides $A$, $\exists$, such as:

**Def:** The notation $\exists ! \ P(x)$ means

**Example:** Show that $\exists x \ (P(x) \lor Q(x))$ and $\exists x \ P(x) \lor \exists x \ Q(x)$ are logically equivalent.

**Need to show:**

**Solution:** Must show 2 things:

(i)

(ii)
How to negate a quantified expression:

How do we negate $\forall x \neg P(x)$?

This illustrates logical equivalence:
Similarly,
These are De Morgan’s Laws for quantifiers.

Example:
1.
2.
3.

Translated into math:

Question: Does #3 follow from #1 and #2?
(We’ll learn how to determine, soon)

1.5 Nested Quantifiers
Two quantifiers are nested if one is in scope of another, e.g.
\[ \forall x \exists y (x - y^2 = 0) \]
Example: Let domain for variables \( x, y, z \) be real numbers \( \mathbb{R} \). The statement

\[
\text{Ex: Let domain for } x, y \text{ be } \mathbb{R}. \text{ The statement}
\]

Note: Order of quantifiers matters!

\[
\text{Ex: Translate the statement}
\]

“\text{The abs value of the product of 2 integers is the product of their absolute values.}”

Let domain be all integers \( \mathbb{Z} \).

\[
\text{Ex of sudoku from last time:}
\]

\[
\begin{array}{ccc}
9 & 9 & 9 \\
\wedge & \wedge & \vee p(i, j, n) \text{ expresses the statement}
\end{array}
\]

\[
i = 1 \quad n = 1 \quad i = 1
\]

“\text{every column contains each integer between 1 and 9.}”

How to write using quantifiers?
§1.6 Rules of Inference

Consider this argument:

1.
2.
3.

Is this a valid argument?

Def: An argument in propositional logic is called . The final one is called if an argument is valid.

To verify that a statement is true, we use

To deduce new statements from statements we already have, we use

Example: Law of detachment or modus ponens is based on tautology:


Let $p$ and $q$ be the prop's
"My cell phone rings" and
"I will disturb the lecture."

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NB: Don't need to memorize these names!!
Ex: Use Rules of Inference to draw a conclusion based on the hypotheses.

Let \( p, q, r \) be “I eat spicy foods,” “I have strange dreams,” “there is thunder while I sleep.” Then hypotheses are:

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<tr>
<th>Step</th>
<th>Reason</th>
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<td>1.</td>
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<td>6.</td>
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Conclusion:

Common mistake (fallacy):
The proposition \([\neg p \land q] \rightarrow p\) is not tautology. (It is false when \( p \) is false \& \( q \) is true). Do not use it in an argument!
Rules of Inference for Quantified Statements

**Universal instantiation:**

Given the premise $\forall x \neg P(x)$, we can conclude that $P(c)$ is true (provided $c$ is an element of the domain).

**Ex:**

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<thead>
<tr>
<th>Rule of Inference</th>
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**Ex:**