Math 55 Lecture 18  §8.1, 8.2 Recurrence Relations

Def: A recurrence relation for the sequence \( \{a_n\} \) is

Ex: Suppose that a person deposits $10,000

Sol: Let \( P_n \) denote

Initial condition:
Ex: Tower of Hanoi.

Goal:

Rules:

Sol.
§8.2 Finding explicit solutions for linear recurrence relations

**Def:** A linear homogeneous recurrence relation of degree $k$ with constant coefficient is

- Linear:
- Homogeneous:
- Constant coeffs:
- Degree $k$: 
$e_n = e_{n-1} + e_{n-2}$

$a_n = a_{n-1} + a_{n-2}$

$h_2 = 2h_{n-1} + 1$

$\beta_n = n \beta_{n-1}$

Approach for finding solution of $e(n)$:

Note: $a_n = r^n$ is a solution

Def: We say $r^k - c_1 r^{k-1} - c_2 r^{k-2} - \cdots - c_k = 0$ is the characteristic equation of $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$, and call

Theorem 1: Let $c_1, c_2 \in \mathbb{R}$. Suppose that $r^2 - c_1 r - c_2 = 0$ has two distinct roots $r_1$ and $r_2$. Then the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ iff
Proof: 2 parts.  

1. Show that if $r_1, r_2$ are roots, and $\alpha_1, \alpha_2$ constants, then

2. Show that if $\{a_n\}$ satisfies recurrence, then
Ex: Find explicit formula for

Sol: 

Note: Theorem 1 doesn't work when char. equation has double root. In this case, use

Theorem 2: Let $c_1$ and $c_2 \in \mathbb{R}$, $c_2 \neq 0$. Suppose $r^2 - c_1 r - c_2 = 0$ has only one root $r_0$. A sequence $\{a_n\}$ is a solution of $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ iff
**Theorem 3**: Let \( c_1, c_2, \ldots, c_k \) be real numbers. Suppose that the characteristic equation \( r^k - c_1 r^{k-1} - \cdots - c_k = 0 \) has \( k \) distinct roots \( r_1, r_2, \ldots, r_k \). Then a sequence \( \{a_n\} \) is a solution to the recurrence relation \( a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k} \) iff

**Ex**: Find solution to

\[
\text{Sol: Characteristic polynomial:}
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Factors: