Def: A random variable is a function from

Note: Random variable is

Ex: Suppose coin

The expected value of a random variable is a

Def: The expected value (or expectation) of the random variable $X$ on the sample space $S$ is
Ex: What is the expected number of times a

Notation: If \( X \) is a random variable on sample space \( S \), let 
\( p(X = r) \) be 

Lemma: If \( X \) a RV w/ range \( X(s) \), \( E(X) \)

Redo Ex above:

Note: If \( f_1 \) and \( f_2 \) are functions from \( A \) to \( IR \), we can
Definition: \((f_1 + f_2)(x) := (f_1 f_2)(x) := \) 

Definition: Given \(a, b \in \mathbb{R}\) and \(f: A \to \mathbb{R}\), we can also define a new function \((a f + b): A \to \mathbb{R}\) by

So if \(X_1\) and \(X_2\) are both random variables with sample space \(S\), we can

And if \(a, b \in \mathbb{R}\), get new RV \(a X_1 + b\) defined by

Theorem "Expectation is Linear": If \(X, X_i, \ i = 1, 2, \ldots, n\) are random variables on \(S\), and if \(a\) and \(b \in \mathbb{R}\), then

(i) 
(ii)
\( Pf (n=2); (i) \, E (X_1 + X_2) \)

(ii)

Ex: Suppose that \( n \) Bernoulli trials are performed, where \( p \) is the probability of success on each trial. What is

\[ \text{Sol:} \]

Def: A random variable \( X \) has a geometric distribution \( w/ \) parameter \( p \) if

Ex: Suppose that the probability that a coin comes up tails is \( p \).
**Lemma:**

\[ \sum_{j=0}^{\infty} x^j = \frac{1}{1-x} \text{ for } |x| < 1 \text{ (limit of geometric series found)} \]

Differentiate both sides:

\[ \sum_{j=1}^{\infty} j x^{j-1} = \frac{1}{(1-x)^2} \]

**Sol:** What is the sample space?

Let \( X \) be random variable equal to

**Theorem:** If the random variable \( X \) has the geometric distribution w/ parameter \( p \), then

...
Def: The random variables $X$ and $Y$ on a sample space $S$ are independent if:

Theorem: If $X$ and $Y$ are independent random variables on a space $S$, then

**Pf:** $E(xy) =$

The expected value of a random variable tells us

What if we want to know how far from the average
Def: Let $X$ be a random variable on sample space $S$. The variance of $X$, denoted $V(X)$, is $V(X) =$

Theorem: If $X$ is a random variable on sample space $S$, then

Using this, can prove...

Thm: If $X$ and $Y$ are two independent random variables on sample space $S$, 