Math 55 Lecture 16 §7.2, 7.3

Last time we informally defined
probability of event =

More formally:

**Def:** If \( S \) is a finite sample space of equally likely outcomes and \( E \) is an event, i.e. a subset of \( S \), then the probability

But what if

**Def:** Let \( S \) be the sample space of an experiment w/o a finite or countable number of outcomes. A probability distribution \( p \) is

Call \( p(s) \) the

**Def:** Let \( S \) be set w/ \( n \) elements. The uniform distribution
Def: If $E \subset S$ is an event, the probability of $E$ is

Theorem: (1) $\Pr(\overline{E}) = \overline{\Pr(E)}$
(2) $\Pr(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{n} \Pr(E_i)$
(3) If $E_1, E_2, \ldots$ is a sequence of pairwise disjoint events in sample space $S$, then $\Pr(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} \Pr(E_i)$

Conditional Probability

Suppose we flip a fair coin 3 times.
Let $F$ be event that
Let $E$ be event that

Suppose we know that $F$ occurs. Given this knowledge, what is

Sol: The only possible outcomes are:

To find conditional prob. of $E$ given $F$, use
Def: Let $E$ and $F$ be events with $p(F) > 0$. The conditional probability of $E$ given $F$, denoted $p(E|F)$, is defined as

Ex: What is the conditional probability

Sol: Let $E = \text{event that}$
Let $F = \text{event that}$

Independence: Suppose a coin is flipped 3 times. Does knowing that the first flip is tails $(F)$ alter the probability that

Note:
Def: The events $E$ and $F$ are independent iff

Ex: Assume that each of the 4 ways a family can have 2 kids

Bernoulli trials

Def: Suppose an experiment has 2 possible outcomes (e.g., coin flip). Each performance of such an experiment called a

Ex: Have biased coin s.t.
Theorem: The prob. of exactly $k$ successes in $n$ indep. Bernoulli trials, w/prob. $p$ of success and prob. $q = 1-p$ of failure, is

$\binom{n}{k} p^k q^{n-k}$

§7.3 Bayes' Theorem

Ex: Have two boxes $B_1, B_2$. 
Sol: Let $E$ be event that
Bayes' Theorem: Suppose that $E$ and $F$ are events from a sample space $S$ s.t. $\Pr(E) \neq 0$ and $\Pr(F) \neq 0$. Then

$$\Pr(F|E) = \frac{\Pr(E|F) \Pr(F)}{\Pr(E)}$$

Ex:

Sol: