(1) Show that if $P$ is a CW poset, then $P$ is graded, i.e. for all $x \in P$, all maximal chains in $[\hat{0}, x]$ have the same length.

(2) Show that the length of an element $w$ in the symmetric group $S_n$ equals the number of inversions, i.e. it is the cardinality of the set $\{(i, j) \mid 1 \leq i < j \leq n \text{ and } w(i) > w(j)\}$.

(3) Show that the Bruhat order on the symmetric group is thin, i.e. every rank 2 interval is a diamond (equivalently every rank 2 interval has precisely 4 elements).

(4) Let $G$ be a finite graph. Let $E$ be the set of all edges in $G$ and let $\mathcal{I}$ be the collection of subsets of $E$ that do not contain all the edges of a cycle of $G$. Use a graph-theoretic argument to show that $(E, \mathcal{I})$ is a matroid (with $\mathcal{I}$ representing the independent sets).