Math 113 Homework 3, due 2/9/2012 at the beginning of class

Policy: if you worked with other people on this assignment, write their names on the front of your homework. Remember that you must write up your solutions independently.

1. Fraleigh, section 6, exercise 32 (as always, carefully justify your answers).

2. Fraleigh, section 6, exercises 44 and 56(a).

3. Let \( n \geq 2 \) be an integer, and let \( \theta = 2\pi/n \). Consider the regular \( n \)-gon \( P \) with vertices \((\cos k\theta, \sin k\theta)\) for \( k \in \mathbb{Z}_n \). The dihedral group \( D_n \) is the set of symmetries of \( P \), which consists of rotations \( r_j \) and reflections \( s_j \) for \( j \in \mathbb{Z}_n \). Here \( r_j \) is the counterclockwise rotation around the origin by angle \( j\theta \), and \( s_j \) is the reflection across the line through the origin and \((\cos j\theta/2, \sin j\theta/2)\). The binary operation is composition.

   (a) Find (and give at least some justification for) general formulas for the products \( r_i r_j \), \( r_i s_j \), \( s_i r_j \), and \( s_i s_j \). For example, \( r_i r_j = r_{i+j} \), where the addition of indices is mod \( n \).

   (b) Show that \( D_n \) is indeed a group.

4. Find all subgroups of \( D_5 \). (Hint: first prove that, if \( H \leq D_5 \) contains a non-trivial rotation, then it contains all rotations.)

5. If \( G \) is a group, the center of \( G \) is defined to be \( Z(G) = \{ x \in G \mid xy = yx \text{ for all } y \in G \} \).

   (a) Show that \( Z(G) \) is a subgroup of \( G \).

   (b) For \( n \geq 3 \), what is the center of \( D_n \)? (Use the multiplication rules you found above. The answer depends on whether \( n \) is even or odd.)

6. How challenging did you find this assignment? How long did it take?