Math 113 Homework 2, due 2/2/12 at the beginning of class

Policy: if you worked with other people on this assignment, write their names on the front of your homework. Remember that you must write up your solutions independently.

1. As explained on page 43 of the book, one can give a seemingly weaker definition of a group, which turns out to be equivalent to the usual one. Namely, consider a set G equipped with a binary operation $*$ such that

- $*$ is associative,
- there exists a *left identity element* $e \in G$ such that $e * x = x$ for all $x \in G$,
- for each $a \in G$ there exists a *left inverse* $a' \in G$ such that $a' * a = e$.

(a) Show that left cancellation holds, namely if $a * b = a * c$ then $b = c$.

(b) Show that the left identity element e is also a right identity element, namely $x * e = x$ for all $x \in G$. (Hint: calculate $x' * (x * e)$ and use part (a)).

(c) Show that the left inverse x' is also a right inverse, namely $x * x' = e$. (Hint: calculate $x' * (x * x')$).

2. (a) Show that multiplication is a well-defined binary operation on the set $\mathbb{Z}_n$ of congruence classes of integers modulo n.

(b) Given an integer $n > 1$, let $\mathbb{Z}_n^*$ be the set of elements $\overline{x} \in \mathbb{Z}_n$ such that there exists $\overline{y} \in \mathbb{Z}_n$ with $\overline{xy} = \overline{1}$. Show that $\mathbb{Z}_n^*$ with the operation of multiplication is a group.

(c) Write multiplication tables for $\mathbb{Z}_8^*$, $\mathbb{Z}_{10}^*$, and $\mathbb{Z}_{12}^*$.

(d) Show that $\mathbb{Z}_8^*$ and $\mathbb{Z}_{12}^*$ are isomorphic, but that $\mathbb{Z}_{10}^*$ is not isomorphic to $\mathbb{Z}_8^*$ and $\mathbb{Z}_{12}^*$.

3. Fraleigh, section 4, exercises 9, 32, 41.

4. Fraleigh, section 5, exercises 13, 51, 54.

5. How challenging did you find this assignment? How long did it take?