Math 113 Homework 1, due 1/26/2012 at the beginning of class

Policy: if you worked with other people on this assignment, write their names on the front of your homework. Remember that you must write up your solutions independently. 1. (a)

Let \( f : X \to Y \) and \( g : Y \to Z \). Show that if \( g \circ f : X \to Z \) is onto, then \( g \) is onto. Show that if \( g \circ f \) is one-to-one, then \( f \) is one-to-one.

(b) Show that \( f : X \to Y \) is bijective (i.e., one-to-one and onto) if and only if there exists \( g : Y \to X \) with \( g \circ f = \text{id}_X \) and \( f \circ g = \text{id}_Y \). (Optional: what happens if you drop one of the two conditions on \( g \)?)

2. Fraleigh, section 0, exercises 29–34.

3. In this exercise we will construct \( \mathbb{Q} \) starting from \( \mathbb{Z} \). Let \( S = \{(a,b)|a,b \in \mathbb{Z}, \ b \neq 0\} \). Define a relation \( \sim \) on \( S \) by

\[
(a,b) \sim (c,d) \iff ad = bc.
\]

(a) Show that \( \sim \) is an equivalence relation.

(b) Let \( \mathbb{Q} \) be the set of equivalence classes, and denote the equivalence class of \( (a,b) \) by \([a,b]\) (ordinarily we would denote this by \( a/b \)). Show that the following operations of “addition” and “multiplication” on \( \mathbb{Q} \) are well defined:

\[
[a,b] + [c,d] = [ad + bc, bd],
\]

\[
[a,b] [c,d] = [ac, bd].
\]

4. Show that every positive integer \( n \) has a binary expansion, i.e. can be expressed as a sum of distinct powers of 2. For example, \( 2010 = 2^{10} + 2^9 + 2^8 + 2^7 + 2^6 + 2^4 + 2^3 + 2^1 \). (Hint: by the division theorem you can write \( n = 2q + r \) with \( r \in \{0,1\} \); use induction.)

Extra credit: show that the binary expansion of a given positive integer is unique.

5. Fraleigh, section 2, exercises 31, 32, 34.


7. How challenging did you find this assignment? How long did it take?