Worksheet 7: Friday 9/22

Acknowledgment: This worksheet has been adapted from that of Gabriel Beiner, a current GSI.

Key Points:
After 9/22 Friday's lecture, you should be able to:

- Understand intuitively and rigorously the definition of continuity

- Apply the Intermediate Value Theorem

Exercises:

1. Prove that the polynomial $x^6 - x^4 - 3x^2 + 2$ has a root.

2. Show that $y = x^3 - 3 + \frac{1}{x}$ has at least two x-intercepts on the interval $(0, 2)$.

3. Suppose $f(x)$ is a function defined on $[0, 1]$ which is continuous except at $x = 1/4$. Let $f(0) = 1$ and $f(1) = 3$. Draw two possible graphs of f, one where $f(x) = 2$ has a solution and one where it doesn't.

4. Suppose that yesterday I left my house in the Berkeley Hills at 7:45AM and walked to my office in Evans, arriving at 9:10AM. Today I took the same route, but left at 8:00AM and arrived at 9:00AM. Prove that there was some place along my route that I passed at the same time on both days (Hint: draw graphs of my positions as a function of time $f(t), g(t)$ on the two days.)