Worksheet 5: Friday 9/15

Acknowledgment: This worksheet has been adapted from that of Gabriel Beiner, a current GSI.

Key Points:
After 9/15 Friday's lecture, you should be able to:

- Calculate harder limits using limit rules
- Apply the ϵ-δ definition to prove basic limit rules

Exercises:

1. Using the ϵ-δ definition of the limit, prove the following theorem.

   Theorem 1. *Let f be a function such that $\lim_{x\to a} f(x) = L$. Then for every constant c, we have that $\lim_{x\to a} cf(x) = cL$.*

2. Find the following limits:

   (a)
   $$\lim_{x\to 8}(1 + \sqrt[3]{x})(2 - 6x^2 + x^3)$$

   (b)
   $$\lim_{x\to \pi} \frac{\sin(x) + x^2 + e^x}{2x}$$

   (c) Let $\lfloor x \rfloor$ denote the largest integer less than or equal to x. Find,
   $$\lim_{x\to -2.4} \lfloor x \rfloor, \quad \lim_{x\to -2^+} \lfloor x \rfloor, \quad \lim_{x\to -2} \lfloor x \rfloor.$$

   (d)
   $$\lim_{x\to 0}(x + x^2)\sin\left(\frac{2\pi}{x}\right)$$

(e)
$$\lim_{x \to 0^+} \ln(\cos(x))$$

(f) $\lim_{x \to -3} f(x)$ where
$$f(x) = \begin{cases} \sqrt{x+3} & x > -3 \\ -2x - 6 & x < -3. \end{cases}$$

(g)
$$\lim_{x \to -2} \frac{2 - |x|}{2 + x}$$

(h)
$$\lim_{x \to 0} \frac{\sqrt{6-x} - 2}{\sqrt{3-x} - 1}$$

(i)
$$\lim_{x \to 0} \frac{2x^2 + 3x + 1}{x^2 - 2x - 3}$$

(j)
$$\lim_{t \to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t}\right)$$

(k)
$$\lim_{x \to 0} \sqrt{|x|} e^{\sin(\pi/x)}$$

(l) $\lim_{x \to 0} f(x)$ where
$$f(x) = \begin{cases} x^2 & x \text{ is rational} \\ -x & x \text{ is irrational} \end{cases}$$