Worksheet 24: Friday 12/1

Acknowledgment: This worksheet has been adapted from that of Gabriel Beiner, a current GSI.

Key Points:
After 12/1 Friday's lecture, you should be able to:

- Use integrals to compute volumes

Exercises:

1. Find the volume of the region enclosed by $y = \sqrt{x}$ rotated around the x-axis for $0 \leq x \leq 1$

2. Find the volume enclosed by $x = 2 - y^2$ and $x = y^4$ rotated around the y-axis.

Final Review:
Here is a list of review problems adapted from Richard Borcherds that should cover most of the topics from the course.

1. Draw a graph of $|\cos(x)|$ for $x \in [-\pi, \pi]$.

2. Evaluate $\lim_{x \to 9} \frac{x^2 - 81}{\sqrt{x} - 3}$.

3. Prove that $x^4 + 1 = 3x$ has a solution.

4. Differentiate $e^x/(x+1)$.

5. Find the derivative of $\cos(\cos(\cos(x)))$.

6. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $x^2y + xy^2 = 2x$.

7. Find the 57th derivative of $\sin(2x)$.

8. If $f(1) = 10$ and $f'(x) \geq 1$ for all x, what is the smallest possible value of $f(5)$?

9. Find $\lim_{x\to\infty} x^{1/x}$.

10. Sketch $y = x\ln(x)^2$ for $x > 0$.

11. Find two numbers whose difference is 10 and whose product is minimal.

12. Using one iteration of Newton's method, estimate $9^{1/3}$ with initial approximation $x_1 = 2$.

13. Find f so that $f'(x) = x^3$ and f is tangent to the line $x + y = 0$.

14. Using a left endpoint Riemann sum with three equal length regions, estimate the area under $f(x) = x^2$ on $[1, 4]$.

15. If $\int_1^5 f(x)\,\mathrm{d}x = 12$ and $\int_1^4 f(x)\,\mathrm{d}x = 14$, find $\int_4^5 (2f(x) + 1)\,\mathrm{d}x$.

16. Prove $1/e \leq \int_0^1 e^{-x^2}\,\mathrm{d}x \leq 1$.

17. Find the derivative of $g(x) = \int_0^x 2te^{-t^2}\,\mathrm{d}t$.

18. Find the derivative of $y(x) = \int_{\cos(x)}^{\sin(x)} \tan(t)\,\mathrm{d}t$.

19. Evaluate $\int_{-1}^1 (x^3 + 2x + 1)\,\mathrm{d}x$.

20. Evaluate $\int_0^{\pi/4} \sec(\theta)\tan(\theta)\,\mathrm{d}\theta$.

21. Evaluate $\int (1 + y)^{10}\,\mathrm{d}y$.

22. Evaluate $\tan(u)\ln(\cos(u))\,\mathrm{d}u$.

23. Evaluate $\int_1^e \frac{\ln(x)^3}{x}\,\mathrm{d}x$.

24. Find the area enclosed by $y = x^2$ and $y = 2/(x^2 + 1)$.

25. Find the volume of the region obtained by rotating the region bounded by the curves $y = \sqrt{x}, y = 0, x = 2, x = 10$, about the x-axis.

26. Find the average value of $\sin(x)^2$ on $[0, 2\pi]$.