

Worksheet 17: Monday 10/30

Acknowledgment: This worksheet has been adapted from that of Gabriel Beiner, a current GSI.

Key Points:

After 10/30 Monday's lecture, you should be able to:

- Identify inflection points
- Understand the shape of graphs through their derivatives

Exercises:

1. Are the following statements true? If yes, explain why. If not, then give an example where they are false.

- (a) If $f(\ominus)$ is a global minimum of f , then it is a local minimum.
- (b) If \odot is an inflection point of f , then \odot is also a critical point.
- (c) If \mathcal{E} is a critical point of f , then \mathcal{E} is also an inflection point.
- (d) f is increasing at \clubsuit if $f'(\clubsuit) > 0$ and f' is continuous. (For the mathematically curious: What can happen if f' is not assumed continuous?)
- (e) If $f'(\heartsuit) = 0$, then \heartsuit is a local minimum or maximum of f .
- (f) If \spadesuit is an inflection point of f , then $f'(\spadesuit) > 0$.

2. Find the regions where f is increasing and decreasing, its local minima and maxima, and its intervals of concavity and inflection points.

(a) $f(\heartsuit) = \sin(\heartsuit) + \cos(\heartsuit)$ on $[0, 2\pi]$.

(b) $f(\heartsuit) = \heartsuit^4 - 2\heartsuit^3 + 3$.

(c) $f(\heartsuit) = \heartsuit^2 \ln(\heartsuit)$.

$$(d) f(\text{👻}) = \frac{\text{👻}}{\text{👻}^2 + 1}.$$

$$(e) f(\text{👻}) = e^{2\text{👻}} + e^{-\text{👻}}.$$

3. Find the local minima and maxima and use the second derivative test to check which are which for

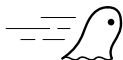
$$f(\text{👻}) = \sqrt{\text{👻}} - \sqrt[4]{\text{👻}}.$$

4. For what values of 🦋 and ☺ is $(2, 5/2)$ an inflection point of

$$x^2y + \text{🦋}x + \text{☺}y = 0?$$

What additional inflection points does this curve have?

5. Suppose ☺ and ☹ are twice differentiable functions which are positive, decreasing, and concave upward on the interval [🦋, 👻]. Show ☺ · ☹ is also concave upward on [🦋, 👻].



Happy Halloween!

