Worksheet 17: Monday 10/30

Acknowledgment: This worksheet has been adapted from that of Gabriel Beiner, a current GSI.

Key Points:
After 10/30 Monday's lecture, you should be able to:

- Identify inflection points
- Understand the shape of graphs through their derivatives

Exercises:

1. Are the following statements true? If yes, explain why. If not, then give an example where they are false.

   (a) If $f(\pumpkin)$ is a global minimum of f, then it is a local minimum.

   (b) If $\skull$ is an inflection point of f, then $\skull$ is also a critical point.

   (c) If $\reversemathwitch$ is a critical point of f, then $\reversemathwitch$ is also an inflection point.

   (d) f is increasing at $\mathbat$ if $f'(\mathbat) > 0$ and f' is continuous. (For the mathematically curious: What can happen if f' is not assumed continuous?)

   (e) If $f'(\mathghost) = 0$, then $\mathghost$ is a local minimum or maximum of f.

   (f) If $\mathwitch$ is an inflection point of f, then $f'(\mathwitch) > 0$.

2. Find the regions where f is increasing and decreasing, its local minima and maxima, and its intervals of concavity and inflection points.

   (a) $f(\reversemathwitch*) = \sin\left(\reversemathwitch*\right) + \cos\left(\reversemathwitch*\right)$ on $[0, 2\pi]$.

   (b) $f(\mathcloud) = \mathcloud^4 - 2\mathcloud^3 + 3$.

   (c) $f(\mathleftbat) = \mathleftbat^2 \ln(\mathleftbat)$.

(d) $f(\text{🧙}) = \frac{\text{🧙}}{\text{🧙}^2+1}$.

(e) $f(\text{👻}) = e^{2\text{👻}} + e^{-\text{👻}}$.

3. Find the local minima and maxima and use the second derivative test to check which are which for

$$f\left(\text{💀}\right) = \sqrt{\text{💀}} - \sqrt[4]{\text{💀}}.$$

4. For what values of 🦇 and 🎃 is $(2, 5/2)$ an inflection point of

$$x^2y + \text{🦇}x + \text{🎃}y = 0?$$

What additional inflection points does this curve have?

5. Suppose 🎃 and 💀 are twice differentiable functions which are positive, decreasing, and concave upward on the interval [🦇, 👻]. Show $\text{🎃} \cdot \text{💀}$ is also concave upward on [🦇, 👻].

Happy Halloween!