Worksheet 12: Friday 10/13

Acknowledgment: This worksheet is adapted from the worksheets made by Amy Dai, themselves being adapted from those of Jeffrey Kuan, former GSIs of this class.

Key Points:
After 10/13 Friday's lecture, you should be able to:

- Compute tangent lines of implicitly defined curves
- Compute derivative of inverse functions using implicit differentiation
- Compute and apply logarithmic derivatives

Exercises:

1. Compute the derivative of $f(x) = x^{(x^x)}$.

2. Find the tangent line to the curve

$$xe^y = x - y$$

at $(0, 0)$. What about at $(a, 0)$ where a is any arbitrary constant?

3. Find the derivative y' of y in terms of x and y for the curve $x^3 + y^3 = 3xy$.

4. The function $y = x^5 + 7x^3 + 10x - 2$ is one-to-one (why?). Assuming this, find the derivative of its inverse using implicit differentiation.

5. We will now try to generalize the computations of the derivatives of $\ln(x)$, $\arcsin(x)$, $\arctan(x)$, etc...

   (a) Suppose f is a one-to-one differentiable function and its inverse function f^{-1} is also differentiable. Use implicit differentiation to show that

   $$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

   provided that the denominator is not 0. (Hint: Use that $f(f^{-1}(x)) = x$).

   (b) If $f(4) = 5$ and $f'(4) = 2/3$, find $(f^{-1})'(5)$.