Worksheet 11: Friday 10/6

Acknowledgment: This worksheet has been adapted from that of Gabriel Beiner, a current GSI.

Key Points:
After 10/6 Friday's lecture, you should be able to:

- Compute trigonometric derivatives
- Apply the Chain Rule

Exercises:

1. Compute the derivative of the following functions:

   (a) $f(\theta) = 2\sec\theta - \csc\theta$

   (b) $g(x) = \frac{1-\sec(x)}{\tan(x)}$

   (c) $h(x) = e^{\tan(x)}$

   (d) $f(x) = \sqrt{x^2 + \sin(x)e^x}$

   (e) $g(x) = (x^2 + e^{2x-1})^3$

   (f) $h(y) = ((3x^5 + e^{2x} + x^4\tan(x))^{12} + 2x)^3$

   (g) $f(\varphi) = \cos\varphi/(1 - \sin\varphi)$

   (h) $g(z) = \cot(z)\cos^2(z)$

   (i) $h(x) = 2^{\sqrt{\sin(x)}}$

   (j) $f(x) = \cot^2(\sin(x))$

   (k) $g(x) = \sin^2(\exp(\sin^2(x)))$

   (l) $h(x) = 2^{3^{4^x}}$

(m) $f(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}$

(n) $g(y) = \sqrt{\frac{1+\sin y}{1+\cos y}}$

(o)
$$h(x) = \sin\left(\frac{e^{x/e^x}\csc(\pi^x)x^{4/5}}{\tan^2\left(12\sin\left(\sqrt{x}^{(1+\sqrt{5})}\right)\right)}\right) \quad \text{Sorry!}$$

2. Find the first and second derivatives of:
   (a) $f(x) = x^4 - 3x^3 + 16x$

   (b) $f(r) = \sqrt{r} + \sqrt[3]{r}$

   (c) $g(y) = 3e^y - 5y$

3. Suppose $f(x) = \sin^2(x)e^{-x}$ and $x(t) = \sqrt{t}/t^2$. Find $f'(x)$ and $x'(t)$. Find
$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} f(x(t)).$$

4. Find the 13th derivative of $f(x) = \cos(2x)$. Find the 5th derivative of x^2e^{4x+3}.

5. For which values of r does $y(x) = e^{rx}$ solve the following differential equation?
$$y'' - 4y' + 3y = 0$$