Worksheet 1: Monday 8/28

Acknowledgment: Worksheets for this class are adapted from the worksheets made by Amy Dai, themselves being adapted from those of Jeffrey Kuan, former GSIs of this class.

Key Points:

- What is a function? What are the different representations of a function?
- What is the domain of a function? What is the range?
- Different properties of a function: even/odd, increasing/decreasing
- Graphing a function
- Modeling using a function

Exercises:

1. Consider the functions $f(x) = 2x + 1$ and $g(x) = x^2 + 2$ defined on $[-2, 2]$:
   (a) What is $f(1)$? What is $g(0)$?
   (b) What is the range of f? What is the range of g?
   (c) Sketch the graphs of f and g on the same coordinate plane.
   (d) Is f even, odd, both even and odd, or neither? Is g even, odd, both even and odd, or neither?
   (e) Where is f increasing/decreasing? Where is g increasing/decreasing?

2. Consider the following functions:

$$f(x) = \frac{x^2 + 2x + 1}{x - 3}$$
$$g(x) = \frac{x^2 - 2x - 3}{x - 3}$$

   (a) What is the domain of f? What is the domain of g?
   (b) For which values of x do we have $f(x) = 0$ and $g(x) = 0$?
   (c) Sketch the graph of $g(x)$.
   (d) Is there a value of x such that $f(x) = 2$? Justify your answer.

3. Find the domain of the following functions:

$$f(x) = \frac{1}{\sqrt{x^2 - 3x + 2}}, \qquad g(x) = \frac{1}{1 - \frac{1}{x-2}}, \qquad h(t) = \frac{\tan(t)}{1 + \sin(t)}$$
$$v(t) = \frac{t}{\sqrt[3]{t^2 - 1}}, \qquad w(t) = \log(t^2 + t - 12)$$

4. Find the range of the following functions defined on $[-1, 1]$:

$$f(x) = x^2 - x + \frac{1}{4}$$
$$g(x) = \exp(-x)$$
$$h(x) = x^3 + x + 3$$

5. (a) If the point $(5, 3)$ is on the graph of an even function, what other point must also be on the graph?
   (b) If the point $(5, 3)$ is on the graph of an odd function, what other point must also be on the graph?

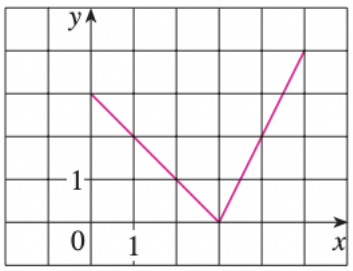
6. (a) What is the parity of the product of two even functions?
   (b) What is the parity of the product of two odd functions?
   (c) What is the parity of the product of an even function and an odd function?

7. Consider the following formulas, which ones can represent a function? Which ones cannot?

$$f(x) = x^3 + \frac{1}{x^3}, \qquad g(x) = e^x$$
$$1 = x^2 + y^2, \qquad 0 = y - \arcsin(x).$$

8. Write piecewise equations for functions that have the following graphs.

49.



50.