Homework 1  
due Thursday, Sep. 7

(1) Suppose $y : [a, b] \to \mathbb{R}^d$ is a continuous function. Show that $\| \int_a^b y(t) \, dt \| \leq \int_a^b \| y(t) \| \, dt$, where $\| \cdot \|$ is any norm on $\mathbb{R}^d$. Hint: one reasonable approach is to break the integral into a Riemann sum and use the triangle inequality:

$$\int_a^b y(t) \, dt = \lim_{n \to \infty} \sum_{k=0}^{n-1} y(t_{kn})h_n, \quad (t_{kn} = a + \frac{k}{n}(b-a), \ h_n = \frac{b-a}{n}).$$

Finish this argument, justifying your steps.

(2) Consider the equation for a pendulum, $y''(t) + \sin(y(t)) = 0$.
(a) Choose initial conditions and calculate one full swing by Euler’s method
(b) Find the frequency of your pendulum
(c) What is the theoretical error bound?
(d) What do you think the error really is?