Homework for Quiz 4 (on October 1)

1. Use the product rule to show that \( \frac{d}{dx} x^n = nx^{n-1} \) for \( n \geq 1 \).
(Here \( x^0 = 1 \) even when \( x = 0 \).)

2. Suppose \( f(x) \) is odd. Prove that \( g(x) = f(x)^n \) is \( \begin{cases} \text{odd if } n \text{ is odd} \\ \text{even if } n \text{ is even} \end{cases} \) for \( n \geq 1 \).

3. Use the fact that \( (f + g)'(x) = f'(x) + g'(x) \) to prove that
\[
\frac{d}{dx} \left( \sum_{i=1}^{n} f_i(x) \right) = \sum_{i=1}^{n} f'_i(x), \quad (n \geq 1).
\]

4. (#39 p. 193) Prove that \( \lim_{x \to \infty} \frac{e^x}{x^n} = \infty \) for \( n \geq 1 \).

5. (#55 p. 196) Show that if \( f(x) = xe^x \), then \( f^{(n)}(x) = (x + n)e^x \) for \( n \geq 1 \).

6. (#52 p. 219) Prove that \( e^x \geq 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} \) for every real number \( x \geq 0 \) and integer \( n \geq 1 \).

7. Show that \( 2^n \geq n^2 \) for \( n \geq 4 \).

8. (#31(c) p. 309) Show that for \( n \geq 1 \),
\[
\int_{0}^{\pi/2} \sin^{2n+1} x \, dx = \frac{2 \cdot 4 \cdot 6 \cdots 2n}{3 \cdot 5 \cdot 7 \cdots (2n+1)}
\]

Sec. 8.1 #1,6,7,8,9,12,13,16,20,23,28,31,32,34,37,38,39,41,45,46