Final Exam, Version A
Math 1A, Spring 2008
Prof. Wilkening

1. (3 points)
(a) state the mean value theorem (for derivatives).
(b) state the mean value theorem for integrals.
(c) state the extreme value theorem.

2. (4 points) Evaluate $\displaystyle\int_0^{\ln\sqrt{3}} \frac{e^x}{1+e^{2x}}\,dx$. Express your answer as a multiple of π.

3. (3 points) Suppose $f(x) = x^3 g(x^2)$. Find $f''(x)$ in terms of g, g' and g''. Simplify your answer as much as possible by collecting like terms.

4. (4 points) Do one step of Newton's method to approximate the solution of

$$1 + x + x^2 + x^3 + x^4 + 2x^5 = 7.026.$$

Use $x_0 = 1$ as the initial guess. Is the result (call it x_1) larger or smaller than the exact solution, x^*, of this equation? Justify your answer.

5. (5 points) Find the volume of the solid obtained by revolving the region bounded by the following curves about the y-axis.

$$y = \frac{1}{1+x^2}, \qquad y = -\sqrt{1-x^2}, \qquad x = 0, \qquad x = 1$$

6. (4 points) Find an explicit formula for the continuous function $f(x)$ such that

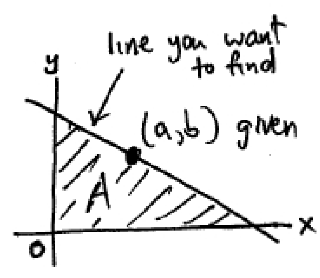
$$\int_0^x f(t)\,dt = x^2 + \int_0^x e^{-t} f(t)\,dt \qquad \text{(for all } x \in \mathbb{R}\text{)}.$$

Don't forget to specify what $f(0)$ is.

7. (6 points) Suppose $f(x)$ is continuous on $[-1, 1]$. Evaluate each of the limits:

(a) $\displaystyle\lim_{x\to\infty} e^{\left(\sqrt{x^2+x} - x\right)}$ (b) $\displaystyle\lim_{x\to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$ (c) $\displaystyle\lim_{x\to\infty} \frac{f(\sin x)}{x}$

8. (5 points) Let $a > 0$ and $b > 0$ be positive numbers. Find the equation of the line through the point (a, b) that cuts off the least area from the first quadrant.



9. (5 points) An elevator starts from rest and accelerates as follows:

$$a(t) = \begin{cases} 0 & 0 \le t \le \pi \\ 2\sin(t-\pi) & \pi < t \le 3\pi \\ 0 & 3\pi < t \le 4\pi. \end{cases}$$

Find the velocity $v(t)$ and the position $s(t)$ for $0 \le t \le 4\pi$ and plot their graphs below.

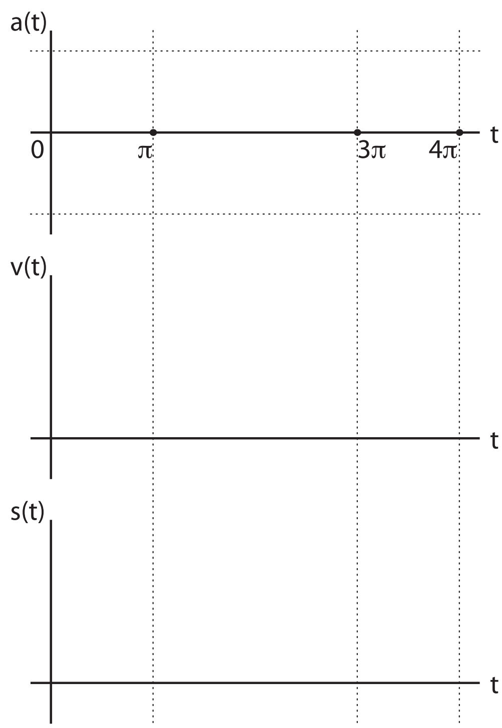
10. (5 points) A cat climbs a telephone pole to catch a squirrel, who escapes by running along the telephone line (which has the shape of a catenary). Their positions (in meters) are given by

$$x_{\text{cat}}(t) = -4 \qquad x_{\text{sq}}(t) = 10\sinh^{-1}\left(\frac{t}{10}\right), \qquad (x_{sq} \text{ stands for } x_{\text{squirrel}})$$

$$y_{\text{cat}}(t) = 2 + t \qquad y_{\text{sq}}(t) = 10\cosh\left(\frac{x_{\text{sq}}(t)}{10}\right) - 5 = -5 + 10\sqrt{1 + \left(\frac{t}{10}\right)^2}$$

Find the rate of change of the distance s between the cat and the squirrel $\boxed{\text{at } t = 0}$.
Hint: $s^2 = x^2 + y^2$ with $x = x_{\text{sq}} - x_{\text{cat}}$, $y = y_{\text{sq}} - y_{\text{cat}}$.

Problem 9



Problem 10