

**Sample Midterm 2**

You are allowed one  $8.5 \times 11$  sheet of notes with writing on both sides. This sheet must be turned in with your exam. *Calculators are not allowed.*

1. (1 point) write your name, section number, and GSI's name on your exam and write your name on your sheet of notes.
2. (4 points) Find the equation of the tangent line to the curve  $y^2 = x^3 + 3x^2$  at the point  $(1, -2)$ .
3. (5 points) Find the relative maxima, minima and inflection points of the function

$$f(x) = xe^{-x^2/2}$$

4. (5 points) Find the function  $u(t)$  that satisfies

$$\frac{du}{dt} = -3(u - 5), \quad u(0) = 1$$

and evaluate  $u(\ln 2)$ .

5. (5 points) Let  $f(x) = \sqrt{4+x}$ . Find the linearization  $L$  of  $f$  at 0 and use the mean value theorem to show that  $f(x) < L(x)$  for  $x > 0$ .
6. (5 points) Evaluate the limit

$$\lim_{x \rightarrow \infty} \frac{\tanh x - 1}{\tan^{-1} x - \pi/2}$$

## Another Sample Midterm 2

1. (1 point) write your name, section number, and GSI's name on your exam and write your name on your sheet of notes.

2. (3 points) Suppose  $f$  is twice differentiable on the interval  $[0, 4]$  and satisfies

$$\begin{array}{cccccc} f'(0) = 1 & f'(1) = 0 & f'(2) = 0 & f'(3) = -1 & f'(4) = 0 \\ f''(0) = -1 & f''(1) = -2 & f''(2) = 0 & f''(3) = 1 & f''(4) = 1 \end{array}$$

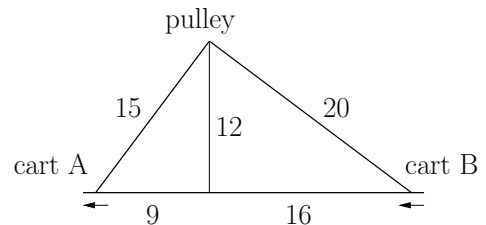
At the endpoints  $x = 0$  and  $x = 4$ , these are one-sided derivatives. Fill in the following table with YES, NO, or CBT (cannot be determined).

$c =$	0	1	2	3	4
$f$ has a local max at $c$					
$f$ has a local min at $c$					

3. (5 points) Let  $f(x) = x^x$ . Compute  $f'(2)$ ,  $f'(4)$  and  $(f \circ f)'(2)$ . Note that  $4^4 = 256$ .

4. (5 points) Use a linear approximation to estimate:  $\frac{1}{\pi} \tan^{-1} \left( 1 + \frac{\pi}{100} \right)$ .

5. (6 points) Two carts are connected by a 35 foot rope that passes over a pulley 12 feet above the floor. Cart A is being pulled to the left at a speed of 2 ft/sec. How fast is cart B moving at the instant cart A is 9 feet from the point on the floor beneath the pulley?



6. (5 points) Show that there is exactly one  $x \in \mathbb{R}$  satisfying

$$x^5 + e^x - 2 = 0.$$

7. (5 points) Do *one* of the following:

(a) Show that

$$\tanh(\sinh^{-1} x) = \frac{x}{\sqrt{1+x^2}} \quad (x \in \mathbb{R}).$$

(b) If  $g(x) = 1 + x + e^x$ , find  $g^{-1}(2)$  and  $(g^{-1})'(2)$ .