Math 156, Fall 2023 Numerical Analysis for Data Science and Statistics

Instructor: Prof. Jon Wilkening

Office: 1051 Evans Hall

Office Hours: Tues 3-4 PM, Fri 3:30-4:30 PM

e-mail: wilkening@berkeley.edu (emergencies & administration only. No questions about HW, please)

online discussion forum for our class: edstem.org, GSI/grader: TBA

Course Announcements, Homework Solutions, etc.: https://bcourses.berkeley.edu/

<u>Lectures</u>: Tues/Thurs 12:40-2:00 PM, 3109 Etcheverry

Required textbook: "Linear Algebra and Learning from Data," by Gilbert Strang.

Recommended reading: "Accuracy and Stability of Numerical Algorithms," by Nicholas J. Higham

"Applied Numerical Linear Algebra," James W. Demmel

"Numerical Optimization," Jorge Nocedal and Stephen J. Wright

Prerequisites: Multivariable Calculus (Math 53) and Linear Algebra (Math 54 or 56)

Syllabus: Introduction to applied linear algebra and optimization with applications in data science. We will cover Parts I, II, V, VI (and a few subtopics of III, VII) of Strang's book, as well as Higham (chap. 2-3), Demmel (chap. 2), Nocedal/Wright (chap. 2-6). In more detail:

- Floating-point arithmetic, relative and absolute error, running error analysis
- Vector spaces (real/complex), bases, dimension, subspaces, linear operators, rank-nullity theorem
- Inner products, adjoint operator, orthogonal projections, closest point property
- Orthogonal and unitary matrices, QR via Gram-Schmidt or Householder, LAPACK implementation
- Eigenvalues and eigenvectors, singular value decomposition (SVD)
- Gaussian elimination, LU factorization, positive definite matrices, Cholesky factorization
- Least squares via QR or the SVD, pseudo inverse, polar decomposition
- Operator norms (1,2,infinity,nuclear norms), linear functionals, Hahn-Banach theorem
- Perturbation theory, condition number, backward stability analysis
- Low rank approximation, Eckart-Young theorem (2-norm, Frobenius norm)
- Discrete probability, sample mean and covariance, principal component analysis
- Advanced calculus review (Jacobian, gradient, Hessian), multivariable Newton's method
- Nonlinear least squares, trust-region framework, Levenberg-Marquardt method
- Line-search methods, Wolfe conditions, conjugate gradients, GMRES, BFGS
- Rank-revealing QR, sparse solutions, deep neural networks, adjoint methods, backpropagation

<u>Grades</u>: Programming assignments: 20% (due Sep 13, Oct 14, Nov 11, Dec 5)

Homework: 20%. 10 assignments, 2 lowest scores dropped

Midterm: 25% (Thursday, Oct 12)

Final: 35% or 60% (Friday, Dec 15, 8-11 AM. Can replace midterm score with the score on the final if helpful.) *No make-up exams for any reason... don't miss the final exam!*

Grade cutoffs: 98 A+, 90 A, 86 A-, 82 B+, 78 B, 74 B-, 70 C+, 66 C, 62 C-, 58 D+, 50 D (no D- given) (raw scores on exams will be mapped to scaled scores, keeping these cutoffs in mind. The scaled score will never be lower than your raw score expressed as a percentage)

<u>Incomplete grades</u>: (University policy) A grade of I will only be given if "your work in a course has been of passing quality but is incomplete for reasons beyond your control"

<u>More details</u>: 10 homework assignments, 4 programming assignments. You may discuss the homework and programming assignments with your classmates, but **you must write up your own solutions**. The two lowest homework scores will be dropped. Python will be the official programming language for the course (submitted to gradescope via a Jupyter notebook.)