

MATH H53 : Mid-Term-2

3rd November, 2015

Name: _____

- You have 80 minutes to answer the questions.
- Use of calculators or study materials including textbooks, notes etc. is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Question	Points	Score
1	10	
2	12	
3	10	
4	10	
5	8	
Total:	50	

1. Consider

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}.$$

(a) (4 points) Show that f is continuous at $(0, 0)$.

(b) (2 points) Find $f_x(0, 0)$ and $f_y(0, 0)$.

(c) (4 points) Find the directional derivative $D_{\mathbf{u}}f(0, 0)$ where $\mathbf{u} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$. Is f differentiable at $(0, 0)$? Justify your answer.

2. (a) (4 points) Find and classify the critical points of

$$f(x, y) = x^3 + y^3 - 3xy.$$

- (b) i. (6 points) Find the absolute maximum of the function $f(x, y) = e^{xy}$ on the curve $x^3 + y^3 = 16$.

- ii. (2 points) Is there an absolute minimum? Why?

3. Let \mathcal{S} be the parallelogram with vertices $(0, 0)$, $(\pi/2, \pi/2)$, $(0, \pi)$, $(-\pi/2, \pi/2)$.

(a) (4 points) Sketch the image of this region under the map $u = x + y$ and $v = y - x$.

(b) (6 points) Evaluate the integral using the above change of variables

$$\int \int_{\mathcal{S}} \sin(x + y) dx dy.$$

4. (a) (2 points) Express the region outside the sphere $x^2 + y^2 + z^2 = z$ and inside the half sphere $x^2 + y^2 + z^2 = 1, z > 0$ in spherical coordinates.

(b) (2 points) What is the volume of the region?

(c) (6 points) Find the centroid of the above region.

5. (8 points) Evaluate

$$\iint_D \max(x^2, y) \, dA,$$

where D is the unit square $[0, 1] \times [0, 1]$.