MATH H53 : Mid-Term-1

22nd September, 2015

Name: ____

- You have 80 minutes to answer the questions.
- Use of calculators or study materials including textbooks, notes etc. is not permitted.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. (10 points) Sketch the curve $r = 1 + 2\cos\theta,\ 0 \le \theta \le 2\pi$ and find the area of the inner loop.

2. (a) (5 points) Find the point of intersection of the lines $\mathbf{r}_1(t) = (1, 0, 2) + t(2, -1, -2)$ and $\mathbf{r}_2(s) = (1, 1, 1) + s(-1, 1, 1)$ or show that they are skew lines.

(b) (5 points) If the lines intersect, find the unique plane containing both the lines. If the lines are skew, find the distance between them.

3. (a) (5 points) Show that the points $P(1,2,2)$, $Q(1,1,1)$, $R(0,2,4)$ and $S(0,-2,0)$ are co-planar.

(b) (5 points) Find the equation of the plane containing all four points.

4. (a) (3 points) Find the parametric equation for the curve of intersection of the plane $2z-x=6$ with the cone $z^2 = x^2 + 3y^2$, and identify (circle, ellipse, hyperbola or parabola) the curve.

(b) (3 points) Find the curvature at $(-2, 0, 2)$.

(c) (4 points) Find the equation for the osculating plane at $(-2, 0, 2)$.

Note: In each of the parts of the following question, to obtain partial-credit, you can assume the statement of the previous part to be true even if you are unable to provide a complete proof.

5. A particle moves in $\mathbb{R}^3$ so that its position vector $\mathbf{r}(t)$ and velocity vector are related by $\mathbf{r}'(t) = \mathbf{b} \times \mathbf{r}(t)$ for some fixed vector $\mathbf{b} \in \mathbb{R}^3$. Suppose also, that $|\mathbf{r}(0)| = 1$.

   (a) (3 points) Show that the particle moves with a constant speed, and that the entire trajectory lies in the unit sphere centered at the origin.

   (b) (4 points) Show that $\mathbf{r}(t) \cdot \mathbf{b}$ is a constant. Using this, show that either the particle is stationary, or it's trajectory is a circle.

   (c) (3 points) If the angle between $\mathbf{r}(0)$ and $\mathbf{b}$ is θ, $0 < \theta < \pi$ show that the particle is not stationary, and calculate the radius of it's circular trajectory in terms of θ.