MATH H53 : ASSIGNMENT-2

(DUE ON 09/14/2015)

- Section 12.3 - 31, 53, 55
- Section 12.4 - 31, 45
- Section 12.5 - 37, 51, 64, 79
- Section 12.6 - 46
- Show that the distance of the origin from the line with parametric equation $\mathbf{r}(t) = \mathbf{r_0} + t\mathbf{v}$ is given by
$$d = |\mathbf{r_0} - \Big(\frac{\mathbf{r_0} \cdot \mathbf{v}}{|\mathbf{v}|^2}\Big)\mathbf{v}|.$$

CHALLENGE : (only for your entertainment, not to be submitted!) - A *tetrahedron* is a polyhedron with four vertices.

(a) Show that a tetrahedron with one vertex at the origin O and the other vertices with position vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ has volume
$$V = \frac{||\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})||}{6}.$$
That is, the volume is exactly one-sixth the volume of any parallelepiped that shares three converging edges with the tetrahedron!
Hint: Use the fact that the volume of a solid is given by
$$V = \int_0^H A(h)\, dh,$$
where $A(h)$ is the cross-sectional area at height 'h'. The calculation should remind you of how one computes the volume of a cone.

(b) Using the above formula, show that volume of a tetrahedron with vertices at $(0, 0, 0), (1, 0, 0), (0, 1, 0)$ and $(0, 0, 1)$ is $1/6$.
We will later give a proof of this fact using triple integrals!

(c) A *regular* tetrahedron, is a tetrahedron with all edges of equal length. Show that the volume of a regular tetrahedron of edge length 'a' is equal to
$$V = \frac{a}{6\sqrt{2}}.$$
Hint: If you place one vertex at the origin, what are the coordinates of the other three vertices so that you get a regular tetrahedron of edge length is 'a'?

Note : The problem numbers are taken from Stewart, Multivariable calculus, Early transcendentals for UC Berkeley, 7th edition.