HOMEWORK-3

- pg 30 - Exercise 25
- pg 31 - Exercise 26
- pg 65 - Exercise 5, 6

Additional problems.

(1) Suppose that $f(z)$ is holomorphic in an open set containing the closed curve γ. Show that

$$\int_\gamma \overline{f(z)} f'(z)\, dz$$

is purely imaginary.

(2) Calculate

$$\int_{|z|=1} (z-1)\, dz, \text{ and } \int_{|z|=1} |z-1|\, |dz|,$$

and verify the triangle inequality.

(3) If $p(z)$ is a polynomial and C_R denotes a circle of radius R around the origin, show that

$$\int_{C_R} p(z)\, d\bar{z} = -2\pi i R^2 P'(0),$$

where we define $d\bar{z} = dx - idy$ and by analogy with complex integration, for $f = u + iv$, we define

$$\int_C f(z)\, d\bar{z} = \int_C (u\, dx + v\, dy) + i \int_C (v\, dx - u\, dy).$$