1. Verify that \( dc \) is a real operator, as follows. Let \( \Omega \subseteq \mathbb{C}^n \) be an open subset, and let \( f: \Omega \rightarrow \mathbb{C} \) be a differentiable function. Express \( dc_f \) in terms of \( \partial f / \partial x_i, \partial f / \partial y_i, dx_i, \) and \( dy_i \). (Do not use the expression for \( dc \) in polar coordinates.)

2. Let \( \mathcal{O}(1) \) denote the metrized line sheaf on \( \mathbb{P}^1_{\mathbb{C}} \) with Fubini-Study metric. Compute the arithmetic intersection number \( (\mathcal{O}(1)^2) \). \[ \textbf{Hint:} \text{ Compute } (D \cdot E) \text{ for suitable arithmetic divisors } D \text{ and } E. \]

3. Find a Green function \( g_\tau \) for the divisor consisting of the point \( P = \tau \) on \( \mathbb{P}^1_{\mathbb{C}} \) (where \( \tau \in \mathbb{A}^1 \subseteq \mathbb{P}^1 \)), subject to the conditions

\[
\begin{align*}
\quad \quad dd^c g_\tau &= \mu \quad \text{and} \quad \int g_\tau \mu = 0, \\
\text{where} \\
\quad \mu &= \frac{dd^c |z|^2}{(1 + |z|^2)^2}.
\end{align*}
\]

\[ \text{Due Tuesday, 7 May} \]