1. Let $k$ be a number field, a function field in one variable, or the completion of such a field at one of its given places. Let $V$ be an affine variety over $k$, and let $i: V \hookrightarrow \mathbb{A}^n_k$ and $j: V \hookrightarrow \mathbb{A}^m_k$ be closed embeddings. Show that for all $c \in k^*$ there is a $c' \in k^*$ such that the set $\{P \in V(k) : P \text{ is integral with respect to } c' \cdot j\}$ contains the set $\{P \in V(\overline{k}) : P \text{ is integral with respect to } c \cdot i\}$.

2. Let $k$ be a field as in the previous problem, let $Y$ be as usual, let $S$ be a finite set of places of $k$ containing $M_k^\infty$, and let $V$ be a variety over $k$. For $i = 1, 2$ let $V_i$ be a complete variety over $k$ and let $X_i$ be a proper model for $V_i$ over $Y$ such that there are compatible open embeddings $V \hookrightarrow V_i$ and $X \hookrightarrow X_i$ such that the complement of the image of $X$ in $X_i$ is the support of an effective Cartier divisor $D_i$ on $X_i$. Let $\lambda_i$ be the partial Weil function over $M_k \setminus S$ for $D_i$ defined using models. You may assume that $X_i$ is normal.

Without reference to integral points or sections of $X_i \rightarrow Y$ or curves in $X_i$, show that $\lambda_1(P) \leq 0 \iff \lambda_2(P) \leq 0$ for all $P \in V(M_k \setminus S)$ (i.e., for all $v \in M_k \setminus S$ and all $P \in V(\mathbb{C}_v)$).