1. Let $k$ be a number field or a function field in one variable over a field $F$, and let $Y$ be as defined in class for $k$. Let $V$ be a variety over $k$, and let $X_1$ and $X_2$ be models for $V$ over $Y$. Let $\mathcal{L}_1$ and $\mathcal{L}_2$ be line sheaves on $X_1$ and $X_2$, respectively, which agree on $V$. Show that $\mathcal{L}_1$ and $\mathcal{L}_2$ agree at almost all places of $k$; i.e., there is a nonempty open subscheme $Y'$ of $Y$ such that $X_1|_{Y'} = X_2|_{Y'}$ (compatible with the isomorphisms of their respective generic fibers with $V$), and such that the restrictions of $\mathcal{L}_1$ and $\mathcal{L}_2$ correspond under this isomorphism.

2. Let $k$ be a number field, a function field in one variable, or the completion of one of these fields at one of their given places. Let $f: V \to W$ be a morphism of complete varieties over $k$. Let $D$ be a Cartier divisor on $V$ whose restriction to the generic fiber of $f$ is effective, and let $\lambda_D$ be a Weil function for $D$. Show that there is a proper birational morphism $g: W' \to W$, a Cartier divisor $E$ on $W'$, and a Weil function $\lambda_E$ for $E$, with the following property. For all $P \in V(M)$ and $Q \in W'(M)$ satisfying $f(P) = g(Q)$, we have $\lambda_D(P) \geq \lambda_E(Q)$.

Edit: Since you haven’t covered blowings-up in Math 256 yet, you may assume that $W$ is nonsingular. (Or, at your option, you may use the following fact. Let $E$ be a proper closed subscheme of $W$. Then there is a proper birational morphism $g: W' \to W$ and an effective Cartier divisor $E'$ on $W'$ such that $g(\text{Supp} E') = E$ (as closed subsets of $W$). Here $g$ is the blowing-up of $W$ along $E$.)