1. Exercise 1 on page 58. You may use standard results on finite fields without proving them.

2(nc). Exercise 2 on page 58. You may use standard results on finite fields without proving them.

[Correction: The book should say \( q = \#\kappa(p) \), not \( q = [\kappa(\mathfrak{p}) : \kappa(p)] \).]

3. Let \( A, K, L, B \) and also \( A, K, M, C \) be as in the “usual picture,” with \( M \supseteq L \) and \( M \) Galois over \( K \). Let \( G = \text{Gal}(M/K) \) and let \( H < G \) be the subgroup corresponding to \( L \). Let \( Q \) be a prime of \( C \) and let \( q = Q \cap B \) and \( p = Q \cap A \). You may assume that the residue field extensions are separable, if necessary, but make it clear when you are using such assumptions.

(a). Express \( G_{Q/q} \) in terms of \( G_{Q/p} \) and \( H \).
(b). Assume that \( L \) is normal over \( K \). Express \( G_{q/p} \) in terms of \( G_{Q/p} \) and \( H \).
(c). Express the inertia group \( I_{Q/q} \) in terms of \( I_{Q/p} \) and \( H \).
(d). Assume that \( L \) is normal over \( K \). Express the inertia group \( I_{q/p} \) in terms of \( I_{Q/p} \) and \( H \).

Here the notation \( G_{Q/p} \) means the decomposition group of \( Q \) in \( \text{Gal}(M/K) \); and \( G_{Q/q}, G_{q/p} \), and corresponding inertia groups are defined analogously.


5. Let \( K = \mathbb{Q}(\mu_3) \) and let \( L = K(\sqrt[3]{2}) \). Determine how the primes \( (2) \) and \( (3) \) in \( \mathbb{Z} \) factor in \( K \) (including the ramification indices and inertia degrees). For each prime of \( \mathcal{O}_K \) over \( 2 \) and \( 3 \), determine how it factors in \( L \) (again, with ramification indices and inertia degrees).

You do not need to give generators for the ideals, just \( e \) and \( f \).