Math 254A. Homework 9

Due Wednesday, 25 October

1. Exercise 1 on page 58. You may use standard results on finite fields without proving them.

2(NC). Exercise 2 on page 58. You may use standard results on finite fields without proving them.

[*Correction:* The book should say $q = \#\kappa(\mathfrak{p})$, not $q = [\kappa(\mathfrak{P}) : \kappa(\mathfrak{p})]$.]

3. Let A, K, L, B and also A, K, M, C be as in the "usual picture," with $M \supseteq L$ and M Galois over K. Let $G = \mathrm{Gal}(M/K)$ and let $H < G$ be the subgroup corresponding to L. Let Q be a prime of C and let $\mathfrak{q} = Q \cap B$ and $\mathfrak{p} = Q \cap A$. You may assume that the residue field extensions are separable, if necessary, but make it clear when you are using such assumptions.

   (a). Express $G_{Q/\mathfrak{q}}$ in terms of $G_{Q/\mathfrak{p}}$ and H.
   (b). Assume that L is normal over K. Express $G_{\mathfrak{q}/\mathfrak{p}}$ in terms of $G_{Q/\mathfrak{p}}$ and H.
   (c). Express the inertia group $I_{Q/\mathfrak{q}}$ in terms of $I_{Q/\mathfrak{p}}$ and H.
   (d). Assume that L is normal over K. Express the inertia group $I_{\mathfrak{q}/\mathfrak{p}}$ in terms of $I_{Q/\mathfrak{p}}$ and H.

   Here the notation $G_{Q/\mathfrak{p}}$ means the decomposition group of Q in $\mathrm{Gal}(M/K)$; and $G_{Q/\mathfrak{q}}$, $G_{\mathfrak{q}/\mathfrak{p}}$, and corresponding inertia groups are defined analogously.

4(NC). Exercise 3 on page 65.

5. Let $K = \mathbb{Q}(\boldsymbol{\mu}_3)$ and let $L = K(\sqrt[3]{2})$. Determine how the primes (2) and (3) in $\mathbb{Z}$ factor in K (including the ramification indices and inertia degrees). For each prime of $\mathscr{O}_K$ over 2 and 3, determine how it factors in L (again, with ramification indices and inertia degrees).

   You do not need to give generators for the ideals, just e and f.