Math 254A. Homework 13

Due Tuesday, 28 November

1(NC). Let K be a complete discretely valued field, and fix an algebraic closure $\overline{K}$ of K (which is a valued field, but is not discretely valued, and may not be complete).

(a). Let L and L' be subfields of $\overline{K}$, finite and unramified over K. Let λ and λ' be the residue fields of L and L', respectively. Show that if $\lambda \subseteq \lambda'$ then $L \subseteq L'$.

(b). Let L_1 and L_2 be subfields of $\overline{K}$, finite and unramified over K. Let λ_1, λ_2, and λ_3 be the residue fields of L_1, L_2, and L_1L_2, respectively. Show that $\lambda_1\lambda_2 = \lambda_3$. (In other words, show that the residue field of the compositum of two finite unramified extension fields of K in some common larger field is the compositum of the residue fields.)

2. Let L/K be a finite extension of complete discretely valued fields, whose residue field extension is separable. Let A and B be the valuation rings of K and L, respectively. Without relying on Exercise 2 on page 52, show that there is a $\theta \in B$ such that $B = A[\theta]$.

3. Show that an infinite algebraic extension of $\mathbb{Q}_p$ is never complete (cf. Exercise 1 on page 134). [**Hint:** Use the last question on Homework 12.]