1(nc). Let $K$ be a complete discretely valued field, and fix an algebraic closure $\overline{K}$ of $K$ (which is a valued field, but is not discretely valued, and may not be complete).

(a). Let $L$ and $L'$ be subfields of $\overline{K}$, finite and unramified over $K$. Let $\lambda$ and $\lambda'$ be the residue fields of $L$ and $L'$, respectively. Show that if $\lambda \subseteq \lambda'$ then $L \subseteq L'$.

(b). Let $L_1$ and $L_2$ be subfields of $\overline{K}$, finite and unramified over $K$. Let $\lambda_1$, $\lambda_2$, and $\lambda_3$ be the residue fields of $L_1$, $L_2$, and $L_1L_2$, respectively. Show that $\lambda_1\lambda_2 = \lambda_3$. (In other words, show that the residue field of the compositum of two finite unramified extension fields of $K$ in some common larger field is the compositum of the residue fields.)

2. Let $L/K$ be a finite extension of complete discretely valued fields, whose residue field extension is separable. Let $A$ and $B$ be the valuation rings of $K$ and $L$, respectively. Without relying on Exercise 2 on page 52, show that there is a $\theta \in B$ such that $B = A[\theta]$.

3. Show that an infinite algebraic extension of $\mathbb{Q}_p$ is never complete (cf. Exercise 1 on page 134). [Hint: Use the last question on Homework 12.]