Math 254A. Homework 11

Due Wednesday, 8 November

1. Prove that $\mathbb{Z}_p$ is compact.

2(NC). Exercise 7 on page 115.

3(NC). Determine the set $\{|a|_p : a \in \overline{\mathbb{Q}}_p\}$. You may assume that the absolute value $|\cdot|_p$ on $\mathbb{Q}_p$ extends to a non-archimedean absolute value $|\cdot|_p$ on $\overline{\mathbb{Q}}_p$.

4. Let K be a number field, let A be its ring of integers, let $\overline{K}$ denote the algebraic closure of K (which is just $\overline{\mathbb{Q}}$), and let $\overline{A}$ be the integral closure of A in $\overline{K}$. Let $\mathfrak{p}$ be a nonzero prime of A. Show that there is a prime ideal $\mathfrak{q}$ of $\overline{A}$ with $\mathfrak{q} \cap A = \mathfrak{p}$. Show also that $\mathfrak{q}$ defines in a natural way a valuation $v\colon \overline{K} \to \mathbb{Q} \cup \{\infty\}$ such that $v\big|_K = \mathrm{ord}_{\mathfrak{p}}$, and that this valuation is not discrete.