Math 254A. Homework 11
Due Wednesday, 8 November

1. Prove that \( \mathbb{Z}_p \) is compact.

2(NC). Exercise 7 on page 115.

3(NC). Determine the set \( \{ |a|_p : a \in \mathbb{Q}_p \} \). You may assume that the absolute value \( |\cdot|_p \) on \( \mathbb{Q}_p \) extends to a non-archimedean absolute value \( |\cdot|_p \) on \( \mathbb{Q}_p \).

4. Let \( K \) be a number field, let \( A \) be its ring of integers, let \( \overline{K} \) denote the algebraic closure of \( K \) (which is just \( \overline{\mathbb{Q}} \)), and let \( \overline{A} \) be the integral closure of \( A \) in \( \overline{K} \). Let \( p \) be a nonzero prime of \( A \). Show that there is a prime ideal \( q \) of \( \overline{A} \) with \( q \cap A = p \). Show also that \( q \) defines in a natural way a valuation \( v: \overline{K} \to \mathbb{Q} \cup \{ \infty \} \) such that \( v|_K = \text{ord}_p \), and that this valuation is not discrete.