Math 115. Homework 5

Due Wednesday 9 October

Section 2.6 (Page 91): 8, 9, 10

Additional problem #1. Prove the following corollary from class on Tuesday, 1 October. (Recall that j was changed to ℓ in the corollary. This was done to avoid notational confusion in the proof).

Corollary. *Let $f \in \mathbb{Z}[x]$, let p be a prime, and let $a \in \mathbb{Z}$. Let $\tau \in \mathbb{N}$ be such that $p^\tau \parallel f'(a)$, and assume that $f(a) \equiv 0 \pmod{p^\ell}$ for some $\ell \ge 2\tau + 1$. Then, for any $\alpha \ge \ell$, there is an integer b, unique modulo $p^{\alpha-\tau}$, such that $b \equiv a \pmod{p^{\ell-\tau}}$ and $f(b) \equiv 0 \pmod{p^\alpha}$.*

For this problem, you may use any results from the textbook (sections 2.6 and earlier), and from the handout on Hensel's lemma *except* the corollary in the handout (which is essentially the same as this corollary).

Section 2.7 (Page 96): 6.

For Exercise 6, prove (by giving counterexamples) that the theorem is false for all composite moduli m.